I. 5. 40

Mr. BLUNDEVIL HIS EXERCISES,

CONTAYNING

Eight Treatiſes,

The Titles whereof are ſet down in the next printed page.

Which Treatiſes are very neceſſary to be read and learned of all young Gentlemen, that have not been exerciſed in ſuch Diſciplines, and yet are deſirous to have knowledge as well in *Coſmographie*, *Aſtronomie*, and *Geographie*, as alſo in the Art of *Navigation*, in which Art it is impoſſible to profit without the help of theſe or ſuch like Inſtructions.

To the furtherance of which Art of *Navigation*, the ſaid Mr. BLUNDEVIL ſpecially wrote the ſaid Treatiſes, and of meere good will doth Dedicate the ſame to all young Gentlemen of this Realme.

The ſeventh Edition corrected and ſomewhat enlarged by Ro. Hartwell *Philomathematicus.*

LONDON,
Printed by *Richard Biſhop*, and are to be ſold by *Benjamin Allen* at the ſigne of the Flowerdeluce in Popes-head Alley. 1636.

EXERCISES

CONTAINING

HIS

M. BLUNDEVILL

THE TITLES OF THE TREATISES contained in this Booke.

First, a very easie Arithmetick, so painely written as any man of a meane capacity may easily learne the same, without the help of any teacher.

Item the first principles of Cosmography, and especially a plaine Treatise of the Spheare, representing the shape of the whole world, together with the chiefest and most necessary uses of the said Spheare.

Item a plaine and full description of both the Globes, as well Terrestriall as Celestiall, and all the chiefest and most necessary uses of the same, in the end whereof are set down the chiefest uses of the Ephemerides of *Iohannes Stadius*, and of certaine necessary Tables therein contained, for the better finding out of the true place of the Sunne and Moone, and of all the rest of the Planets upon the Celestiall Globe.

Item a plaine and full description of *Peter Plancius* his universall Map, lately set forth in the yeere of our Lord 1592, containing more places newly found, as well in the East and West Indies, as also towards the North Pole, which no other Map made heretofore hath, whereunto

is alſo added how to finde out the true diſtance betwixt any two places on the Land or Sea, their longitudes and latitudes being firſt knowne, and thereby you may correct the Scales or Tronkes that be not truly ſet downe in any Map or Card.

Item, a briefe and plaine deſcription of M. *Blagraue* his Aſtrolabe, otherwiſe called the Mathematical Iewell, ſhewing the moſt neceſſary uſes thereof, and meeteſt for Sea-men to know.

Item, the firſt and chiefeſt Principles of Navigation more plainely, and more orderly taught than they have beene heretofore by ſome that have written therof, lately collected out of the beſt modern Writers, and Treaters of that Art.

Item, a briefe deſcription of univerſall Maps and Cards, and of their uſe: And alſo the true uſe of *Ptolomie* his Tables, together with the true order of making the ſaid Tables. And of all other Maps and Cards as well univerſall as particular, and that according to the doctrine of the beſt Geographers that be, or have beene in theſe later dayes.

Item, the true order of making of *Ptolomie* his Tables, and alſo the making of all other Tables, Maps or Cards, as well univerſall as particular, and that according to the doctrine of the beſt Geographers that be or have beene in theſe later dayes.

And moreover, I have thought good to adde unto mine Arithmeticke, as an appendix depending thereon, the uſe of the Tables of the three right lines belonging to a circle, which lines are called Sines, lines Tangent, and lines Secant, whereby many profitable and neceſſarie concluſions, as well of Aſtronomy, as of Geometrie are to be

wrought

wrought onely by the helpe of Arithmetick, which Tables are set down by *Clavius* the Iesuite, a most excellent Mathematician, in his Booke of demonstrations made upon the Spheriques of *Theodosius*, more truly printed than those of *Monte Regio*, which Booke whiles I read at mine own house, together with a Loving Friend of mine, I tooke such delight therein, as I minde (God willing) if God give me life, to translate all those propositions, which *Clavius* himselfe hath set downe of his owne, touching the quantity of Angles, and of their sides, as well in right line Trangles, as in the Spherical Triangles, of which matter, as *Monte Regio* wrote diffusedly, and at large, so *Copernicus* wrote of the same briefly, but therewith somewhat obscurely, as *Clavius* saith. Moreover, in reading the Geometry of *Albertus Durerus*, that excellent Painter, and finding many of his conclusions very obscurely interpreted by his Latine Interpreter (for hee himselfe wrote in high Dutch) I requested a friend of mine, whom I knew to have spent some time in the study of the Mathematiques, not only plainely to translate the foresaid *Durerus* into English, but also to adde thereunto many necessary propositions of his owne, which my request hee hath (I thanke him) very well performed, not only to my satisfaction, but also to the great commodity and profit of all those that desire to be perfect in Architecture, in the Art of Painting, in free Masons craft, in Ioyners craft, in Carvers craft, or any such like Art commodious and serviceable in any common Wealth, and I hope that he will put the same in print ere it bee long, his name I conceale at his owne earnest intreatie, although much against my will, but I hope that hee will make himselfe known in the publishing of his Arithmeticke,

metick, and the great Art of *Algebra*, the one being almost finished, and the other to be undertaken at his best leasure, as also in the Printing of *Durerus*, unto whom he hath added many necessary Geometricall conclusions, not heard of heretofore, together with divers other of his workes, as well in Geometry, as in other of the Mathematicall Sciences, if hee be not called away from these his studies by other affaires. In the meane time I pray all young Gentlemen and Sea-men to take these my labours already ended in good part, whereby I seeke neither prayse nor glory, but only to profit my Country.

To

To the Reader.

I Greatly rejoyce to see many of our English Gentlemen, both of the Court and Country in these dayes, so earnestly given to travell as well by Sea as Land, into strange and unknowne Countries, and specially into the East and West Indies, following therein the good example of divers worthy Knights and Gentlemen, that have ventured their lives to discover strange Countries to the great honor of their Country and to their owne immortall fame. And because that to travell by Sea requireth skil in the Art of Navigation, in which it is impossible for any man to be perfect, unlesse he first have his Arithmetick, and also some knowledge in the principles of Cosmography, and spetially to have the use of the Sphearе, of the two Globes, of the Astrolabe, and Crosse-staffe, and such like instruments belonging to the Art of Navigation, I thought good therefore to write the Treatises before mentioned, to serve as an introduction for such yong Gentlemen as have not been exercised in such kind of studies, which Treatises if they shall vouchsafe to read with attentive minde, and in such order as they are before set down, I doubt not but that it will cause them hereafter to seeke for further knowledge therein. And in any wise I wish them to begin with my Arithmetick, the contents whereof are declared in the next Chapter following. In the meane time I do earnestly request all yong Gentlemen, to take these my simple Pamphlets no lesse thankfully than they have done my Horse-booke, and in so doing, I shall have just cause to thinke my labour well bestowed.

What

What cauſe firſt moved the Author to write this Arithmetick, and wish what order it is here taught, which order the contents of the Chapters thereof hereafter following do plainly ſhew.

I Began this Arithmetick more than ſeven yeeres ſince for a vertuous Gentlewoman and my very deare friend Mrs. Elizabeth Bacon, the daughter of Sir Nicholas Bacon Knight, a man of moſt excellent wit, and of moſt deepe iudgement, and ſomtime Lord Keeper of the great Seale of England, and lately (as ſhe hath beene many yeers paſt) the moſt loving and faithfull wife of my worſhipfull friend M. Iuſtice Windham, not long ſince deceaſed, who for his integrity of life, and for his wiſedome and juſtice daily ſhewed in government, and alſo for his good hoſpitality, deſerved great commendation. And though at her requeſt I had made this Arithmeticke ſo plaine and eaſie as was poſſible (to my ſeeming) yet her continuall ſickneſſe would not ſuffer her to exerciſe her ſelfe therein. And becauſe that divers having ſeene it, and liking my plaine order of teaching therein, were deſirous to have Copies thereof, I thought good therefore to print the ſame, and to augment it with many neceſſary Rules, meet for thoſe that are deſirous to ſtudy any part of Coſmographie, Aſtronomie, or Geographie, and ſpecially the Art of Navigation, in which without Arithmeticke, as I have ſaid before, they ſhall hardly profit. But now to returne to my matter, the contents of this

this Arithmeticke are these here following. First, having defined what Arithmeticke is, and what numeration or numbring is, and of what parts it consisteth, and how to number any great summe written in many figures, I deale with the foure speciall kinds of Arithmetick, that is, Addition, Subtraction, Multiplication, and Division. Then I shew the order of working in whole numbers called in Latine Integra, by the rule of proportion, otherwise called the golden rule or the rule of three; Then I treat of broken numbers called fractions, setting down seven necessary rules, belonging thereunto, by the helpe whereof you shall be the better able to adde, subtract, to multiply, and divide the same. That done, I shew how to use the rule of three, as well in dealing with sole Fractions, as with Fractions annexed to Integrums, which rule of three is threefold, that is, the cõmon rule, the rule reverse, & the double rule, the order of all which three kinds, I do plainly teach by examples, shewing therein, how and when they are to be used. Next to those I set down the rule of fellowship, a necessary rule for those that have to traffick in any trade of Merchandise, giving divers examples thereof. Next to that I treat of Arithmetical & Geometricall Progression, and also of Proportion, and of the three kinds thereof, that is, of Proportion Arithmetical, Geometricall, and Musicall. Then I shew how to finde out the square root of any number, and also the use thereof, in setting of Battels, and also how to finde out the cubique root of any number: and last of all I treat of Astronomicall fractions, shewing how to adde, to subtract, to multiply, and divide the same, and also to take the square root thereof, without the knowledge of which Fractions, you can never calculate any thing truly out of the Astronomicall Tables.

Some perhaps do looke here that I should speake somwhat of the rules of Algebra, wherby all subtle and intricate questions

of

of Arithmetick are to be unfolded, wherwith I leave indeed to deale, partly for that I have not of long time exercised my selfe therin, and partly because I know that one hath begun to write thereof, whose Booke being once ended, I doubt not but that he will shortly after print they same to the great profit and furtherance of all those that delight in such good exercises. But in the meane time I have thought good to adde (as I have said before) unto this Arithmeticke a plaine description, together with the use of the Tables of Sines, of Lines Tangent and Secant, which Tables will pleasure many that would gladly know how, and wherein to use them, and specially such Seamen as have some taste of Arithmeticke, without the which no good almost is to be done in any Science.

The Treatise of Arithmeticke containeth 26 Chapters, as followeth.

WHat Arithmeticke is, what Numeration is, and of what parts it consisteth, and what signification every Digit hath according to his place, and how to expresse or tell a great number written in many figures, *Chap.* 1

Of the foure speciall kindes of Arithmeticke, and first of Addition, with examples thereof, *Chap.* 2

Of Subtraction, with examples thereof, and how to try the same *Chap.* 3

Of Multiplication and certaine Tables belonging thereunto, together with the use thereof, and what is to be observed therein, with examples and tryall thereof, *Chap* 4

Of Division, and what is to be observed therein, with examples and tryall thereof, and of halfing any number, *Chap.* 5

Of the rule of three, called the Golden rule, and what order is to be observed in working thereby, and of the three kinds thereof, *Chap.* 6

Of Fractions what they be, with a demonstration thereof, together with seven necessary rules belonging to the same, and what every rule teacheth, *Chap.* 7

Of Addition, Subtraction, Multiplication, and Division of Fractions, *Chap.* 8

Of the common rule of three belonging to Fractions, with examples, *Chap* 9

Of the rule reverse, called in Latine, *Regula reversa*, and the order of working thereby, with examples, *Chap.* 10

Of the double rule called in Latine, *Regula duplex*, and the order of working thereby, with examples, and in working thereby how to know when you have to use *Regula reversa*, or the common rule of three, *Chap.* 11

Of the Rule of Fellowship, and the order of working thereby, with divers examples thereof, *Chap.* 12

Of Progression, what it is, and of the two kinds thereof, that is, Arithmeticall and Geometricall, *Chap.* 13

Of

Of Addition belonging to Progreſſion Arithmeticall, and the order thereof, with examples, *Chap.*14

Of Addition belonging to Progreſſion Geometricall, and the order thereof, with examples, *Chap.*15

Of Proportion, what it is, and of the three kinds thereof, that is, Arithmeticall, Geometricall, and Muſicall, *Chap.*16

Of Arithmetical Proportion, what it is, and how it is divided, *Chap.*17

Of Proportion Geometricall, what it is, and how it is divided, *Chap.*18

Of the chiefe and ſpecial kinds of Geometrical Proportion, that is, of equality and inequality both greater and leſſer, *Chap.* 19

Of Proportion of the greater inequality, and of the two kinds thereof, that is, *Simplex*, and *Multiplex*, and of their divers kinds, with certaine tables belonging thereunto, *Chap.*20

Of Proportion of the leſſer inequality, *Chap.*21

Of Muſicall Proportion, what it is, and of the two kinds thereof, that is, Simple and Compound, of which Compound there be alſo two kinds, that is, proper and unproper, *Chap.*22

How Proportions are to be ſet down in writing, and how they are to be added, ſubtracted, multiplyed, and divided, even like to fractions in all reſpects, *Chap.* 23

How to finde out the ſquare root of any number, with examples, *Chap.*24

The uſe of the ſquare root in ſetting of Battels, which according to the Italian uſe are to bee ſet foure manner of wayes, the order whereof is here ſet down with examples. *Chap.*25

How to finde the Cubique root of any number, and the order thereof with examples, *Chap.*26

Of Aſtronomicall Fractions, whereto they ſerve, and how to adde, ſubtract, multiply, and to divide the ſame, *Chap.*27

How to divide ſuch fractions when the diviſor is greater than the dividend, *Chap.*28

How to take the ſquare root of Aſtronomicall Fractions, *Ch.*29

And immediately after theſe Chapters do follow the uſe of the Tables of Sines, lines Tangent, and lines Secant, before mentioned, together with the Tables themſelves.

Mr. BLUNDEVIL

HIS

EXERCISES.

CONTAYNING

Eight Treatiſes.

The firſt Treatiſe.

CHAP. I.

Of Arithmetick.

WHat is Arithmetick?

It is the Art of counting or numbring by figures.

What is to number by figures?

It is to expreſſe the value of any number in his proper Characters and figures, which is called by a Latine name Numeration.

What belongeth to Numeration?

Two things, to know the ſhapes of the figures, and ſignification of their places.

How many figures are there?

Theſe ten, 1,2,3,4,5,6,7,8,9,0, whereof the tenth made like an o, as you ſée héere, is called a Cypher, which is no number of it ſelfe, but ſerveth only to fill up a number.

What is number?

Number

Number is a collection or summe of many ones or unities added together.

How is number divided?

Into three kinds; that is, Digit, Article, and Compound.

Which be they?

The Digit is any of the first nine Figures before set downe. Article is any number ending in a Cypher, as 10 is one Article, 20 is two Articles, 30 is three Articles, &c. Compound is that which is compounded of Article and Digit, as 13, 14, 17, 24, &c.

Shew what signification every Digit hath according to his place, and in what order such place is to be considered in expressing any number?

The order as touching the place, is to beginne at the right hand, and so to proceed towards the left hand. For any of the nine Digits whatsoever, standing in the first place, which is on the right hand, signifieth the value of himselfe only, in the second place tenne times himselfe, in the third place an hundred times himselfe, in the fourth place a thousand times himselfe, in the fifth place tenne thousand times himselfe, in the sixth place an hundred thousand times himselfe, in the seventh place a million, or thousand thousand times himselfe, in the eighth place ten millions, in the ninth place an hundred millions, in the tenth tenth a thousand millions, &c.

Doth the Cypher signifie nothing?

Alone it signifieth nothing, otherwise it maketh a place whersoever it standeth, so as it be not the outermost on the left hand: for there it hath no place at all, as heere you may see in this number 04500 whereof the first Cypher on the right hand, signifieth the first place, the second Cypher the second place, but the outermost Cypher on the left hand, signifieth no place at all, because it hath no Digit standing before it towards the left hand, and therefore though in this number there be five figures, yet it signifieth no more than if there were but foure, viz. foure thousand and five hundred.

By what meanes may a great number written in many figures, be readily expressed or told?

By dividing the same into divers parts with strokes or pickes

prickes made at the end of every third figure, beginning to tell from the right hand towards the left as in this number: 4|320|570 In which, beginning with the Cypher on the right hand, I tell one, two, and three, and there make a streeke, and so proceeding forth still towards the left hand, I make a streeke at the end of every third figure, by which streekes or partitions I make them now generall numbers, and every streeke may be named by his severall denomination. Notwithstanding, in expressing this number being thus divided, or any other such like, you must begin at the left hand, and say in this Example thus, foure Millions, three hundred twenty thousand, five hundred and seventie: for by reason of this Division, the Figure 4 standeth here alone, and in the first place, and indeed signifieth foure Millions, and by that meanes, you may fitly expresse the said number in saying as afore, thus, foure Millions, three hundred and twentie thousand, five hundred and seventie. And the better to discerne the Millions from the rest in great numbers, it shall not be amisse to set an M signifying Millions, right over the head of the streeke which is drawne betwixt the sixth and seventh Figures, |34|545 M |678|694, which is to be uttered thus, thirty foure thousand, five hundred, forty five Millions, five hundred seventy eight thousand, six hundred ninety and foure crownes or pounds, or whatsoever other denomination or name it shall please you to give them.

Chap. II.

Of the foure speciall kinds or parts of Arithmeticke.

Which are those foure kinds?

These, Addition, Subtraction, Multiplication, and Division.

Why is not Numeration also counted as a part?

Because Numeration together with the figures and places whereof it consisteth, are counted rather as first Elements, and Principles of Arithmeticke, then as parts or speciall kinds thereof.

Of Addition.

WHat is Addition?

It is that which teacheth to bring many seuerall summes into one summe.

How is that done?

First, by placing euery seuerall number one right under another, under which you must draw a Line: that done, you must adde together the numbers of the first ranke, beginning on the right hand with the lowest Figure of the same ranke, and so going upward to the highest Figure of the same ranke, and so from ranke to ranke, till you come to the last, and if the summe of any ranke do not exceed the number of any of the foresaid nine Digits, then set downe that Digit which comprehendeth that number under his proper ranke, beneath the Line, but if the summe of that ranke exceedeth the number of any one Digit, by reason that it consisteth of Articles and Digits, then set downe the Digit, and keepe the Article or Articles in your minde, to be added to the first Figure of the next ranke of the left hand, but if the summe be an euen Article or Articles, then set downe a Cypher, keeping the number of Article or Articles in your mind, be it one, two, or three, to be added to the next ranke: all which things you shall better vnderstand by this example here following. As for Example, I spent in one yeare 125 l, in another yeare 234 l, and in another yeare 240 l. Now to know the totall sum of all this, I place these seuerall summes one right under another, and then I draw a Line under them as here you see.

	l.
	125.
	234.
	240.
	599.

Then beginning on the right hand with the lowest Figure of the first ranke above the Line, I say that a Cypher and 4 is but 4. Againe, 4 and 5 maketh 9, which I set downe under the Line, then proceeding to the second ranke towards the left hand, I say that 4 and 3 maketh 7, and 7 and 2 maketh 9, which also I set downe, then removing to the third ranke, I say that 2 and 2 maketh 4, and 4 and 1 maketh

maketh 5, which I also set downe as you see in the former example, so that the totall summe under the line is 599 l.

Another example having Cyphers mixt with digits.

Here I say that 9 and 8 maketh 17, and 17 and 7 maketh 24, wherefore I set downe the digit 4, and keepe two Articles in mind, which being added to the lowest figure of the second ranke, which is 4, maketh 6, then 6 and 4 maketh 10, here I set downe a Cypher, keeping one Article in mind, which being added to the figure 5 of the third ranke maketh 6, which I also set downe, then I say that 3 and 4 maketh 7, and 7 and 3 maketh 10, for the which I set downe first a Cypher: and then because there is no more to bee added, I set downe on the left hand the one Article which I had in mind, so as the whole summe commeth to 10604, as in the former example.

	3047.
	4508.
	3049.
	10604.

How are pounds, shillings, pence, halfe-pence, and farthings, and all other numbers of divers Denominations to be added?

You must divide every severall name into divers columnes or spaces by themselues, and then beginning with the first on the right hand, you must adde every columne by it selfe, bringing farthings to halfe-pence, and halfe-pence to pence, pence to shillings, and shillings to pounds, setting the summe of every columne under the nether line, as you see in this example following.

l	s	d	ob.	q;
345	13	1	0	1
234	11	0	1	1
45	14	9	1	0
320	6	8	1	1
946	5	8	0	1

Here first beginning with the columne of farthings, I find therein 3 farthings which is one halfe-penny and one farthing. Wherefore I set downe the odde farthing as you see, and keepe the halfe-penny in mind: then adding the halfe-penny in mind to the lowest halfe-peny of the second columne, I say that 1 in mind and 1 maketh 2, and 2 and 1 maketh 3, then 3 and 1 maketh 4, which 4 halfe-pence, because they make just two pence, I set downe a Cypher, keeping the two pence in mind, which two pence being added to 8 maketh 10, then 10 and 9 maketh 19, and 19 and 1 maketh 20. Now because that 20 d maketh 1 shilling and 8 d, I set downe

the 8 ß kæping the shilling in mind, which 1 shilling being added to the 6 of the next columne maketh 7, then 7 and 4 maketh a 11, and 11 and 1 maketh 12, then 12 and 3 maketh 15, wherefore I set downe 5, kæping the Article in mind which being added to 1 of the next coloumne maketh 2, and 2 and 1 maketh 3, and 3 and 1 maketh 4 Articles, which 4 Articles maketh 40 ß, which is two pound, which I kæpe in mind, and therefore I adde the 2 ℔ to the columne of pounds, saying that 2 and 5 maketh 7, and 7 and 4 maketh a 11, and a 11 and 5 maketh 16, wherefore I set downe 6 kæping the 1 Article in mind, which being added to 2 of the next columne, maketh 3, then 3 and 4 maketh 7, and 7 and 3 maketh 10, then 10 and 4 maketh 14, wherefore I set downe 4, kæping 1 Article in mind, which being added to 3 of the next columne maketh 4, then 4 and 2 maketh 6, and 6, and 3 maketh 9, which I also set downe, so as the totall summe amounteth to 946 ℔, 5 ß 8 ð no halfe-penny, one farthing, as you sæ in the former example.

How shall I know whether these severall summes bee truely added or not?

Some do teach it to be done by culling out all the nines, which way is more tedious then sure: for the surest tryall indæd is to be done by subtracting the severall summes out of the totall sum, of which subtraction wee come now to speake, for all the foure speciall kinds are tryed one by another.

Chap. III.

Of Subtraction.

What doth Subtraction teach?

It teacheth to take a lesser number out of a greater, and to sæ what remayneth.

What is to be obserued in this kind?

First, you must set downe your greater number above, and then the lesser number right under the same. As for example, I have lent to one 564 pounds, and hee hath paid me thereof 57 pounds. Here to know what remaineth, I first

first set downe the number lent, and under that the number paid, and then drawe a line as you see in this manner.

Lent.	564. l.
Paid.	57. l.

Here beginning on the right hand, I first say, take 7 out of 4, that cannot be, wherefore I take one Article of the next figure or place of the lent number, which Article being added to 4 maketh 14, then I say, take 7 out of 14 and there remaineth 7, which I set downe under the 4, then I adde that one Article which I borrowed, to the second figure of the paid number which is 5, saying that 5 and 1 in mind maketh 6, then take 6 out of 6 and there remaineth nothing, wherefore I set downe a cypher under the 5 of the paid number, then I proceed to the third figure of the lent number, which is 5, and because I find nothing written under it, nor having in mind to take out of it, I say, take nothing out of 5, and there remaineth still 5, so as the remainder is 507 pound, as you see in this example following.

Lent.	564. l.
Paid.	57.
Remain.	507.

How shall I know whether this bee right or not?

By adding the remainder and the number paid together, the summe whereof (if you have done well) will be all one with the number lent, as in the former example, I first adde 7 and 7 together, and that maketh 14, wherefore according to the precepts of Addition before taught, I set downe 4, keeping the Article in mind, then I say, 1 in mind and 5 maketh 6, which I also set downe, then I say nothing and 5 is 5, which I set downe in the third place, which in all maketh 564, a number equall to the number lent, as you see here following.

Lent.	564. l.
Paid.	57.
Remaine.	507.
Proofe.	564.

You may perceive by this, that if any figure of the paid number be greater than the figure over him, out of the which it is to be subtracted, you must alwayes borrow one Article of his next fellow, to be added againe to him in his proper place. But you have to note, that having to deale with numbers of divers denominations, then in borrowing any

number, you must alwayes have respect to the denomination or name of the thing from whence you borrow, as in borrowing from shillings you borrow 12 pence and not 10, from pounds you borrow not one Article, but 2 Articles which do make 20 s, but when the whole number is altogether of one selfe denomination, then you must alwayes borrow one Article which is 10, to make up your number that wanteth. As you shall more plainly perceive by this example containing numbers of divers denominations, as of pounds, shillings, and pence, halfe-pence, and farthings. Suppose therefore that you have lent to one 467 l, 13 s, 4 d, ob, q; and he hath paid you againe thereof 89 l, 16 s, 9 d, ob, q. Here having set downe the summe lent in severall columnes, according to their divers names, and then the sum paid: right under the same draw a line as you see in this Example following.

	l	s	d	ob	q
Lent.	467	13	4	1	1
Paid.	89	16	9	1	0
Remaine.	377	16	7	0	1

Here beginning with the first columne on the right hand, I say take nothing out of one, and one still remayneth, which I set downe, then proceeding to the next, I say take one out of one and nothing remaineth, wherefore I set downe a Cypher, then proceeding to the next columne, I say take 9 out of 4, that cannot be, wherefore I borrow a shilling of the next columne, that is 12 d, which being added to 4, maketh 16 pence, then I say take 9 out of 16, and there remaineth 7, which I set downe, then proceeding to the next, I adde the one shilling which I borrowed, to the 16 which maketh 17, then I say take 17 s out of 13 s, that cannot be, wherefore I borrow one pound which is two Articles of the next ranke, that is 20 s, which being added to the 13 s, maketh 33 s, then I say take 17 out of 33 s and there remaineth 16 s, which I set downe; then I adde the one pound which I borrowed to 9, and that maketh 10, then I say take 10 out of 7, that cannot be, wherefore I borrow one Article out of the next 6, which being added to the 7 maketh 17, then I say take 10 out of 17 and there remaineth 7, which I set downe, then the one which I borrowed, I adde to the 8 of the next ranke, and that maketh 9. Againe, I

say

ſay take 9 out of 6, that cannot be: wherefore borrowe one Article of the next 4, which being added to 6 maketh 16, then I ſay take 9 out of 16 and there remaineth 7, which I ſet downe, then I take the one which I borrowed out of the 4, and there remaineth 3, ſo that the remainder is as you ſee in the former example, 377 ℔, 16 ſ, 7 d, no ob, one q;.

How ſhall I try whether this be true or not?

By adding the remainder and the ſumme paid together, as in the former example: and of that Addition, will riſe, if you have done truly, a ſumme like in every condition to the ſumme lent. In making which proofe or tryall you cannot likely erre, if you remember to reduce pence to ſhillings, and ſhillings to pounds, and therefore in the columne of pence, no particular ſumme can bee above 11 d, nor in the columne of ſhillings no particular ſumme can bee above 19 ſ, for if it be 20 ſ then it is a pound, and muſt be brought to the columne of pounds.

Chap. IIII.

Of Multiplication.

What is Multiplication.

It is the producing or bringing forth of a third number, by multiplying two other numbers the one into the other: And it conſiſteth of three numbers, that is, the Multiplicand, the Multiplyer, and the Product.

What ſignifie thoſe names?

The multiplicand is that number which is to be multiplied, and the multiplier is that whereby the ſame is multiplied, and the Product is the ſumme of ſuch multiplication: As for example, if I would multiply 4 by 3, as in ſaying three times 4 maketh 12, here the number of 4 is the multiplicand, and the number 3 is the multiplyer, and the number 12 is the Product of that multiplication.

What order is to be obſerved in multiplying, and how are thoſe numbers to be ſet?

Before I teach you the true order of multiplying, I thinke it good to set you downe a Table of Multiplication, which unlesse you learne perfectly by heart, you shall never multiply readily nor quickly.

The first Table of Multiplication.

2	2	4	3	5	15	4	9	36	6	8	48	9	9	81
2	3	6	3	6	18	4	10	40	6	9	54	9	10	90
2	4	8	3	7	21	5	5	25	6	10	60	10	10	100
2	5	10	3	8	24	5	6	30	7	7	49			
2	6	12	3	9	27	5	7	35	7	8	56			
2	7	14	3	10	30	5	8	40	7	9	63			
2	8	16	4	4	16	5	9	45	7	10	70			
2	9	18	4	5	20	5	10	50	8	8	64			
2	10	20	4	6	24	6	6	36	8	9	72			
3	3	9	4	7	28	6	7	42	8	10	80			
3	4	12	4	8	32									

How is this Table to be read?

In this manner: 2 times 2 maketh 4, and 2 times 3 maketh 6, and 2 times 4 maketh 8, and so forth, multiplying still one Digit by another, untill you come to 100, for this Table serveth only for Digits, which may bee made to extend so farre as you will, and untill you have learned the foresaid Table without Booke, you may help your selfe with this other Table of digits, made squire-wise as you see here

1	2	3	4	5	6	7	8	9	0
1	2	3	4	5	6	7	8	9	1
	4	6	8	10	2	14	16	18	2
		9	12	15	18	21	24	27	3
			16	20	24	28	32	36	4
				25	30	35	40	45	5
					36	42	48	54	6
						49	56	63	7
							64	72	8
								81	9

In the Front of which Table are set downe the 9 Digits, beginning on the left hand, & so proceding to the right hand, from 1 to 9. Againe on the right side of the said table are set down

the

the foresaid 9 digits, beginning above and so proceeding right downe from 1 to 9, the use whereof is thus; first seeke the digit tobee multiplyed in the front, and seeke the other whereby you have to multiply the same on the right hand, & the square angle answering to these 2 digits, will shew the Product of such Multiplication. As for example, having to multiply 7 by 6, I first seeke the greater digit which is 7 in the front, & the lesser which is 6 on the right hand, the product whereof I find in the square angle answering both to 7 and to 6 to be 42, and the like is to be observed in any 2 digits that are of the like value, as 7 times 7, the Product whereof is 49. But the first table being perfectly learned without booke, whensoever you have to multiply one number by another, you must observe these rules here following.

First, that you set downe the first figure of your multiplyer right under the first figure of the number that is to be multiplyed, on the right hand, and then orderly to place the rest of the figures of your multiplyer, be they few or many, towards your left hand, directly under the rest of the figures of the number that is to be multiplyed, for if the figures stand not in order one right over another, it will breed a confusion in your working. Secondly, you must not forget to multiply all the figures of the number that is to be multiplyed, by the first figure of your multiplier, before you deale with the next multiplyer, beginning alwayes on your right hand, and so to proceed from one to another, whereby you shall make as many severall Products as there be figures in your multiplyer. Thirdly, you must remember to set downe the first figure of every severall Product, right under the figure of that multiplyer whereby you do multiply, and having ended your multiplication, draw a line under the severall products: that done, adde the severall products together according to the rules of Addition: and the summe thereof shall be the totall or generall Product of that multiplication: all which rules you shall the better understand by working this example following. Suppose then that you would know how many houres there are in a yeare, knowing first that a yeare consisteth of 365 dayes and 6 houres, here because that every naturall day containeth 24 houres comprehending both day and night, you have to multiply 365 by 24,

and

and therefore you must first set down 365, because it is the greater number, and is the multiplicand, which must alwayes stand above, and right under that, your multiplyer which is 24, is to be set downe in his due place, according to the Rules before taught, thus, as you see here.

Then say thus, 4 times 5 maketh 20, and having set downe a cypher right under the 4, keep the 2 articles in mind, then say 4 times 6 is 24, and 2 in mind is 26,

365, the multiplicand.
24 the multiplyer.

here set downe 6 and keepe 2 in mind, then say 4 times 3 is 12, which with the 2 in mind maketh 14, here first set downe the 4 under the 3, and because you can proceed no further, you must therefore set downe hard by the 4 on the left hand the one Article which you had in mind, then having cancelled the first figure of the multiplyer, by making a dash thorow it with your Pen, as you see in the example following: Proceed with the other figure of the multiplyer, saying that two times 5 maketh 10, wherefore set downe a Cypher right under the said 2 keeping the one Article in mind, then say 2 times 6 is 12, and one in mind maketh 13, wherefore set downe 3, and keepe one in mind, then say 2 times 3 is 6, which with one in mind maketh 7, the which you must set downe: and because you have made an end of your multiplication, cancell the 2, and draw another line under the 2 severall Products; that done, adde together whatsoever is contained betwixt the two lines, and you shall find the generall Product to be 8760 houres, unto which adding the odde 6 houres, the number of houres contayned in the whole yeare will be 8766 houres.

How shall I know whether the last multiplication be right or not?

By dividing the generall Product by the multiplyer, for in so doing your Quotient will be like unto the first summe that was multiplyed, which you cannot do, untill such time as you have learned to divide, and therefore having first shewed certaine compendious wayes of multiplication, I will then proceed to division.

The multiplicand,	365.
The multiplyer,	24.
The severall Products,	1460. 730.
The generall Product or total summe	8760.

Certaine

Certaine compendious wayes of Multiplication.

WHen is any such way to be used?

When the multiplyer beginning on the right hand with one Cypher or with many, endeth on the left hand with the Digit, as these Numbers following, 10|100|1000 &c.

What is then to be done?

If you have to multiply by 10, then you have no more to do, but to set downe on the right hand of the number that is to bee multiplyed, one Cypher, if by a 100, then two Cyphers, if by a 1000, then 3 Cyphers, as for example; if you would multiply 365 by 10, then by setting downe on the right hand one Cypher, as hath beene said, the Product will be 3|650, if you set downe two Cyphers, the Product will be 36|500, if you set downe three Cyphers, then the Product will be 365|2000.

What if the number do end on the left hand with any other Digit, as 2, 3, or more, as 200, 300, 400?

Then you must multiply the number of the multiplicand, first, by that Digit, and then adde to the end of the Product on the right hand all the Cyphers annexed to the said Digit, as if you would multiply 365 by 200. First, say 2 times 5 maketh 10, and 2 times 6 maketh 12, and 1 in mind is 13, and so to proceed in multiplying every figure of the multiplicand by 2, and you shall find the Product to be 730, whereunto if you adde on the right hand 2 Cyphers, the whole Product will be 72|000.

CHAP. V.

Of Division.

WHat is Division?

Division is that whereby any number is divided into as many parts as you will.

How many numbers are incident to Division, and how are they called?

These foure, that is to say, the number which is to be divided, which is called the Dividend, the second the number, whereby

whereby you do divide, which is called the Divisor, the third number is called the Quotient, which sheweth how many times the Divisor is comprehended in the Dividend, and the fourth number is called the Remainder, if any be.

What order is to be observed in Division?

This here following, first set downe your Dividend, and directly under that, beginning on the left hand, set downe your Divisor, that is to say, the first figure of your Divisor right under the first figure of your Dividend on the left hand, and so consequently one after another proceeding towards the right hand, which is alwayes to be done so often as your Divisor doth not exceed in quantity, the figure standing right over his head: for if it do, then you must remove your Divisor one Figure further towards the right hand. As for Example; if you would divide 487 by 53, you must not set the first Figure of your Divisor which is 5, under the first Figure of your Dividend, which is but 4, but under the second Figure of your Dividend, which is 8, for you cannot take 53 out of 48, and therefore you must set the first Figure of your Divisor under the second Figure of the Dividend, and so follow on with the rest, and then draw a line, as you see here in this Example following.

The dividend. 487.
What is thereto be done? The divisor. 53. (9. the quotient.

Then you must aske how many times 5 is comprehended in 48, and you shall find that 5 is comprehended in 48, 9 times, which 9 being the Quotient, must be placed on your right hand, behind a crooked line made like a halfe Moone, as you see in the Example above, then multiply the first Figure of the Divisor by the quotient 9, and the product thereof shal be 45, which being taken out of 48 there will remaine 3, which you must set down over the head of 8, and streeke out the 48, and also the first Figure of your Divisor which is 5, that done, multiply the second Figure of your Divisor which is 3, by the foresaid Quotient which is 9, and the Product thereof will be 27, which being taken out of 37 there remaineth 10, which you must set over the head of 37, and cancell the 37 and all the rest beneath, as you see in this Example following.

Wherein

Wherein you see that the dividend is 487, the Diuidor 53 the quotient 9, and the remainder 10, which if you will apply to any use, you may imagine that there is 487 l. to be divided amongst 53 Souldiers, and by working as before, you shall find that every Souldier must have nine l. and there shall be remaining ten l. to bee divided amongst the foresaid 53 Souldiers, which ten l. being reduced to shillings, may be divided amongst the Souldiers, as well as the pounds: As for Example, if you multiply 10 l. by 20 s. it will make 200 s. which being divided by 53. the first Divisor, the quotient will be 3 s. and the remainder 41 s. and that being reduced into pence, which is done by multiplying 41 by 12, the quotient will be 9 d. and the remainder 15 d. which if you multiply by 4, you shall reduce them to farthings, the product whereof will be 60, which product being divided by 53, the first Divisor, you shall find in the quotient one farthing, and the remainder to bee $\frac{7}{53}$ of a farthing. the value of which Fraction how to find out is taught hereafter when wee come to speake of Fractions. So as you see by this meanes that every Souldier shall have for his share 9 l. 3 s. 9 d. q; and somewhat more, a thing of no moment.

30	
487	(9, the quotient.
53	

How many things are to be remembred in Division.

These five Rules here following. First, that you put no number at one time in the quotient above 9. Secondly, to moderate your quotient in such sort, as having multiplied the first figure of the Divisor by the quotient, there may remaine sufficient for the next figure of the Divisor being multiplyed by the quotient, to bee deduced out of that number which standeth right over his head. Thirdly, that you multiply every figure contained in the Divisor by the quotient. Fourthly, that if at any time in working it happeneth so as your Divisor is not comprehended in the number over his head, then to put a Cypher in the quotient, and to remoove your Divisor one figure further towards the right hand, and that done to worke as before. Fifthly, to see that the last remainder, if there be any left, do

not

not exceed in quantity the Divisor, all which things you shall better understand by this one Example following: Suppose then that you have to divide 819096 by 92, here having set downe your Dividend and Divisor in such order as is before taught, and as you see here in this Example.

First aske how many times 9 is in 81, and you shal find the quotient to be 8, which being multiplyed into 9 maketh 72, which if you make out of 81, there shall remayne 9, which must be set downe over the head of figure 1, and cancell the said 81, together with the first figure of the Divisor, which is 9, then say two times eight is sixteene, which being subtracted out of 99, there remayneth 83, which you must set over the head of the 99, and cancell the 99 that is above, and also the 2 beneath, as you see in the former Example. That done, remove your Divisor towards the right hand, that is to say, by setting the last figure of your Divisor which is 2, under the next Cypher on the right hand, and place the first figure of your Divisor which is 9, next to that towards the left hand: then aske how many times 9 is comprehended in 83, and you shall find 9 times, which 9 must be set downe in the quotient next to the 8, then say 9 times 9 is 81, which being taken out of 83, there remaineth 2, then having set downe the 2 over the head of 3, cancell the 83 above, and also the 9 beneath, then multiply the last figure of the Divisor which is 2 by the 9 which is in the quotient, and the Product thereof is 18, which being taken out of 20, there remayneth 2, which 2 must be set downe over the Cypher, and the Cypher cancelled, and also the 2 beneath, as you see in this Example following.

		2
That done, remove your Divisor one figure further towards the right hand by setting the 2 under the 9, which is the last		832
	The dividend.	819096 (89.
	The divisor.	92

figure

figure saue one of the Dividend, and place the first figure of your Diuisor which is 9, next to that on the left hand, then aske how mane times 9 is contained in 2, and you shall find none, wherefore you must set downe a Cypher in the quotient, and cancell the Diuisor, as you see in this example.

That done, remove your Diuisor againe by setting the 2 under the last figure of the dividend, and place the 9 next to that on the left hand, then aske how many times 9 is contayned in 29, and you shall find 3 times, which 3 you must set downe in the quotient, as you see in the example following, then multiply the first figure of your Diuisor which is 9, by 3, and the Product thereof will bee 27, which being taken out of 29, there will remaine 2, which 2 you must set over the head of 9, and cancell the 9 above, together with the 9 beneath, then multiply the 3 which is in the quotient by the last figure of your Diuisor which is 2, and the Product thereof will bee 6, which being taken out of 29 there remaineth 20, wherefore you must set down a Cypher over the 6, which is the last figure of the Dividend, and cancell the 6 above, and also the 2 beneath, and then you have done rightly, and in such order as all the former examples and this also next following do plainly shew.

		the remainder.	
		2 20	20.
The Dividend.	8 1 9 0 9 6	(8903	92.
The Diuisor.	9 2	the quotient.	

For here you see that the dividend is 819096. the diuisor 92. the quotient 8903. & the remaynder 20, which remaynder must bee set on the right hand of the quotient over the Diuisor, having a line drawne betwixt them as you see in the last example.

How shall I know whether I have divided truely or not?

By multiplying the quotient by your Diuisor, and by adding to the Product thereof the remaynder, for so shall you have a number like to the first Dividend if you have wrought well, if not, then your summe will be either more or lesse than the Dividend.

Certaine

Certaine compendious wayes of Division.

IS there no shorter way of Division.

No, unlesse the Diuisor hath one Cypher or more on the right hand, for then you may use a briefer way by cutting off the last figure, or figures of the dividend: As for example, if you had to divide 3708, by 10, here cut off the last figure 8 with a dash of your Pen in this wise 370|8 and the quotient will bee 370, and the remainder shall be 8. Againe, if you had to divide the foresaid number by 100, then cut of the two last figures of dividend, that is, the 8, and the Cypher standing next it, and then the quotient will be but 37, and the remainder 8, as before, but if you haue to divide any number by 1000, then you must cut of three figures of the dividend, and so forth, remembring alwaies for every Cypher in the Diuisor, to cut off one figure or Cypher of the dividend.

Of halfing any Number.

HOw is that done?

By dividing the number by 2. As for example, if you would know the halfe of 3708, divide the same by 2, and you shall find the quotient to be 1854, which is the just one halfe of the dividend.

CHAP.

Chap. VI.

Of the Rule of proportion, otherwise called the golden Rule, or Rule of Three, whereof there be three kinds, that is, The common Rule Direct, the Rule Reverse, and the Double Rule.

WHerefore is it called the Rule of Three.

Because that by three knowne numbers, it teacheth you to find out a fourth number unknowne.

How are such three numbers to be placed?

The first is to bee placed on the left hand, the third on the right hand, and the second in the middest betweene both.

How shall I know in what place each one is to be set?

By marking to what number the question is annexed, for that must alwayes bee the third number, and having the third number, you shall quickly have the first, because the first and third must alwayes bee of one selfe Denomination or name, betweene which two, the second number is to bee placed: As for example, let this bee the question: If I pay 35 s for 13 yards of linnen cloath, how much shall I pay for 3 yards, heere because the question is annexed to the number 3 yards, that must be the third number, and is to be placed on the right hand, then because that the number 13 yards is of the selfe-same Denomination, that must be the first number, and is to be placed on the left hand, and the number 35 s (which is the price) is to bee placed in the middest betwixt the other two, as you see here following.

Now must you make your question in this manner, if 13 yards did cost me 35 s, what shall 3 yards cost me? and so the first and last shall be of one Denomination or name, that is to say, yards.

yards.	s.	yards.
13.	35.	3.

What order is to be observed in working by this Rule?

You must multiply the third and second numbers the one into the other, and divide the Product thereof by the first, and the Quotient will shew you what the fourth number should bee, which fourth number is alwayes of like Denomination to the second or middle number, as in the former example: first multiply 35 by 3, and the Product will be 105, which if you divide by 13, the Quotient will be eight shillings, and one third of a shilling which is foure pence, and that is the price of three yards which is the number that you seeke to know. And this is the common kind of working by the Rule of three, whereof it is called the common Rule of three.

What if there be divers Denominations either of coyne, as of pounds, shillings, and pence; or parts of yards, as quarters, halfe-quarters, and such like?

Then you must reduce them all to the smallest Denomination which belongeth to Fractions, whereof wee come now to treate.

Chap. VII.

Of Fractions.

What are Fractions?

They are broken parts of some whole thing called in Latine Integrum.

What is *Integrum?*

Any thing that is whole, and not broken, or divided into parts: as one whole yard, a pound, a shilling.

Of how many numbers doth a Fraction consist?

Of two, that is, the Numerator, and the Denominator, whereof the Numerator is alwayes set above, and the Denominator beneath, having a little line drawne betwixt them thus $\frac{1}{2}$ which signifieth one second or one halfe; againe, two thirds and three fourths are written thus $\frac{2}{3}$ $\frac{3}{4}$ whereof the first signifieth two thirds, and the other three fourths and such like.

What

What is meant by these two words, Numerator and Denominator, and whereto serve they?

The Denominator is as much to say as the namer of the parts, which sheweth into how many parts the Integrum is to be divided, and the Numerator is as much to say as the numberer, and sheweth how many of those parts are to be taken: As for example, $\frac{2}{3}$ of a shilling; here the nethermost called the Denominator, sheweth that the shilling is to be divided into three parts, as into three groats, and the upper number called the Numerator, sheweth that you must take 2 out of those 3 parts, so as $\frac{2}{3}$ of a shilling is as much to say, as two groats or 8 d. Againe, fractions may bee divided into two kinds, that is, into simple fractions, and into fractions of fractions.

Define those two kinds?

Simple fractions are such as I speake of before, which are the first and immediate parts of any integrum that is divided into parts, but if those fractions be divided into more fractions, then are they called fractions of fractions, as when I say three fourths of two thirds, of six sevenths, and such like.

How are such fractions to be set downe in writing?

In manner of simple fractions thus, $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$.

Make some Demonstration of this example, that I may the better understand it?

Imagine that there is a whole peece of gold of 7 s, as our angell in times past hath beene, which 7 s bee the seven parts of that peece of gold, for tryall whereof lay downe before you 7 twelue-penny peeces of silver, or in stead thereof 7 counters, to signifie those seven parts, of which seven parts you must first take away six, rejecting or laying aside the odde seventh part, then divide those 6 parts into three parts, and every such part will be just 2 s, of which three parts you must take away two parts, that is 4 s, rejecting the other third part which remayneth, that done, divide those two parts which you have taken away into foure parts, which is 4 s, and take three of them, rejecting the fourth part, so shall you find that $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of the forelaid peece of gold is just 3 s, and there remaineth still of the parts rejected 4 s, which being added to 3 that was taken away, do make

up again the whole integrum of 7 ā. Notwithstanding fractions of fractions do seldome chance in the division of any number, but if they do, you must reduce them into simple fractions, before you can deale with them any manner of way, and because there are divers Rules belonging to fractions, without the knowledge whereof you can neither adde, nor Subtract them, nor yet ioyne them with any integrum, I will briefly set downe here seven necessary Rules for the same.

Seven necessary Rules belonging to Fractions.

WHat doth the first Rule teach?

To bring Fractions of Fractions into proper simple Fractions.

The first Rule,

How is that done?

Thus; multiply the first numerator into the second, and if there be any more fractions, then multiply also the said Product of the first two numerators into the third numerator, and that shall be the numerator of the simple fraction: then multiply the denominators in like manner, and the summe thereof shall be the denominator of the simple fraction. As in the former example ½ of ⅓ of ⅙, here I multiply the first two numerators together, in saying two times three maketh six, now because I have a third numerator, I multiply the Product of the first two numerators, which is 6, into the third numerator which is also 6, saying 6 times 6 maketh 36, and that shall be the numerator of the simple fraction, which I set downe. Then by multiplying the three denominators in like manner, I find the denominator of the simple fraction to be 84, the which I set downe under the 36, and draw a line betwixt them, so as now I find that ½ of ⅓ of ⅙ do make $\frac{36}{84}$ which indeed is no more but ⅜ as you shall learne hereafter by the first Rule.

What doth the second Rule teach?

To bring improper fractions being more in value then integrums into integrums.

When are fractions said to be more then integrums?

When the numerator is equal or a greater number then the

denominator,

minator, so it they be both of like value as ½ ⅔ and such like, then such fraction is an even Integrum.

How shall I know how many *Integrums* such fractions as be more then *Integrums* do containe?

By dividing the Numerator by the Denominator, and if any thing remaine, write that above the Denominator, as in this example 115⅐, here if you divide 808 by 7 which is the Denominator, you shall find the Quotient to containe 115 Integrums, and the remainder to be 1, which is ⅐.

What doth the third Rule teach?

To bring Integrums into parts by multiplying the number of the Integrums by the Denominator of those parts, as if you would bring 64 yards into quarters, multiply 64 by 4, and there will arise thereof 256 quarters.

What doth the fourth Rule teach?

It teacheth to bring Integrums having fractions annexed to them, into one fraction.

How is that done?

Multiply the number of the Integrums by the Denominator of the fraction annexed, and then adde to the Product the Numerator of the said fraction, and that summe shall be the Numerator, under which write the Denominator aforesaid, and so you shall find that 23 Integrums having ⅘ annexed thereunto shall make ¹¹⁹⁄₅.

What doth the fift Rule teach?

It teacheth to expresse a fraction written with many Figures, in so few as may be.

How shall I do that?

Thus; find out some number that may first divide the Numerator, and then the Denominator severally by themselves, without leaving in either of them any remainder, and the Quotient of the first division will shew the Numerator, and the Quotient of the second division will shew the Denominator, but if you cannot readily find out a number that will divide them both without leaving some remainder, then leave not to subtract the lesser number out of the greater, until you find two like numbers, by one of the which two like numbers divide the Numerator,

and also the Denominator severally as before, and the Quotient will shew that which you seeke: but if such two like numbers cannot be found, then you may assure your selfe, that the fraction cannot be written in fewer figures than they already be.

Give example of these two wayes?

For example of the first way, suppose that you would expresse $\frac{9}{12}$ in lesser numbers, here seeke out some number that may divide evenly both the Numerator, and also the Denominator, without leaving in either of them any remainder, which by dividing each of them by the number of 3 you may do: For by asking how many times 3 is contained in 9, the Quotient will be 3, and by asking how many times 3 is contained in 12, the Quotient shall be 4, so shal you find $\frac{3}{4}$ to be no more then $\frac{9}{12}$. Now for example of the second way, let $\frac{27}{81}$ be the fraction which you would set downe in fewer figures, here because you cannot readily find out a number that will evenly divide both the Numerator and Denominator, subtract the lesser out of the greater, that is to say 27 out of 81, and there will still remaine 54, from which subtract 27, and there will remaine 27, which two numbers because they are both alike, divide the Numerator and Denominator of the foresaid fraction, each of them by one of these numbers, that is to say, by 27, and the Quotient of the first division will shew you the Numerator which is 1, and the Quotient of the second division will shew you the Denominator which is 3, so as you shall find $\frac{1}{3}$ to be as much in value as $\frac{27}{81}$ and note that when there bee cyphers both in the Numerator and the Denominator standing in such sort as they may bee evenly cut off, the remainder will shew the fewest figures, wherein the fraction may bee written as $\frac{300}{700}$ here by cutting off the two cyphers as well beneath as above the line, with a dash of your Pen in this manner $\frac{3}{7}\frac{00}{00}$ the remainder shall bee $\frac{3}{7}$ which is as much in value as $\frac{300}{700}$. And fractions may be set downe in minimis terminis, that is to say, in the least numbers more readily by division, then by subtraction in this manner: First divide the greater number by the lesser, and if there be any remainder left, then by that remainder divide the last Divisor

that

that went next before, continuing still so to do, untill nothing remaine, or you find such a number as will evenly divide the last Divisor, without leaving any remainder, for that number will divide both the Numerator and also the Denominator of the given fraction, without leaving any remainder. As for example, suppose the given fraction to be $\frac{36}{15}$ which you would set downe in minimis or smallest numbers: here having divided 36 by 15, you find the remainder to bee 6, by which 6 you have to divide againe the last Divisor which is 15, and you shall find the remainder to be 3, which doth evenly divide 6, without leaving any remainder at all. Wherefore you may boldly say that 3 is the common Divisor, as well to the Numerator as to the Denominator of the given fraction, for if you divide 36 by 3 you shall find in the Quotient 12, which shall be the Numerator of the new found fraction, and by dividing the Denominator of the given fraction, which is 15 by 3, you shall have in the Quotient 5, and that shall bee the Denominator of the new found fraction, so shall the new found fraction bee $\frac{12}{5}$ which is in value as much as the given fraction $\frac{36}{15}$. And you have to note that such numbers as may be evenly divided by another number without leaving any remainder, are called Compounds. But such as cannot bee divided but that there will remaine some odde unite, those are called Primes. You have also to understand that if the given fraction be such as the greater number thereof may at the first be evenly divided by the lesser number, without leaving any remainder, then the Quotient of that first division shall be the common Divisor. As for example, let $\frac{27}{81}$ bee the given fraction, as it was in the first example that was to bee wrought by subtraction, here by dividing the greater number by the lesser, you shall find no remainder but onely 3 in the Quotient, which is the common Divisor, for if you divide the Numerator of the given fraction 27, by 3, the Numerator of the new found fraction will bee 9, and having divided the Denominator of the given fraction which is 81 by 3, the Denominator of the new found fraction will bee 27, which is no more in value but $\frac{1}{3}$. for if you divide 9 by 3, the Quotient will bee 3, and if you divide 27 by 3,

the Quotient will be 9: againe, by dividing 9 by 3, the Quotient will be 3, then aske how many times 3 is contained in 3, and there will be but 1, and by asking how many times 3 is contained in 9, you shall find 3, so as the new fraction shall bee $\frac{1}{3}$, and thus you see that $\frac{3}{27}$ or $\frac{1}{9}$ is either of them but $\frac{1}{3}$ and that the number 3 which was the first Quotient, is the common Diviſor as well to the Numerator as to the Denominator of the first given fraction. And looke as compound fractions may be ſet downe in minimis by diviſion, ſo may compound proportions be ſet downe in minimis by like manner of diviſion, as ſhall bee declared hereafter when we come to ſpeake of proportions.

What doth the ſixth Rule teach?

It teacheth to find out the value of the fraction of any Integrum.

How is that done?

Thus; multiply the Numerator by the next leſſer knowne parts of the Integrum, and divide the Product thereof by the Denominator, ſo ſhall you have the value of the fraction. As for example, if you would know the value of $\frac{1}{4}$ of an angell, conſider first what parts an angell hath, and you ſhall find the parts thereof to be 10 s. or 30 groats, here if you multiply 3 by 10 s. it will make 30 s, which being divided by 4, you ſhall find in the Quotient 7 s. and the remainder to be $\frac{2}{4}$ or one halfe of a ſhilling, which is 6 d. Againe, if you multiply 3 by 30 groats, it will make 90 groats, which being divided by 4, you ſhall find in the Quotient 22 groats, and the remainder to be $\frac{2}{4}$ or one halfe of a groat which is 2 pence, making in all 7 s. 6 d, like unto the firſt ſum, ſo as you ſee that $\frac{1}{4}$ of an angell is 7 s. 6 d, and thus you may deale with the fractions of any other Integrum that hath knowne parts.

What doth the ſeventh Rule teach?

It teacheth to bring fractions of divers denominations to one ſelfe-denomination, without the which neither addition, nor ſubtraction of fractions can be made: As for example, if you would adde $\frac{1}{3}$ and $\frac{2}{4}$ together, you muſt firſt bring them to one ſelfe-denomination thus: multiply the Denominators the one into the other, and the Product thereof ſhall bee a common Denominator to both the fractions, wherefore ſay 3 times 4 doe

make

make 15. which must be set downe in two severall places by themselues thus, 15 and 15 Then multiply the Numerator of the first fraction into the Denominator of the second, as 2 into 5 maketh 10, which set downe over the common Denominator thus $\frac{10}{17}$ then multiply the Numerator of the second fraction into the Denominator of the first fraction as 4 into 3, whichmaketh 12, and that must bee likewise set downe over the second common Denominator thus $\frac{12}{17}$ so shall you find that $\frac{2}{3}$ and $\frac{4}{5}$ are all one in value with $\frac{10}{17}$ and $\frac{12}{17}$.

Chap. VIII.

Additions of Fractions.

How are fractions to be added together?

Fractions being first brought to one Denomination are easily added, for then you have no more to do but adde the Numerators together, and to write the common Denominator under the summe of such addition, as in the former example $\frac{10}{17}$ being added to $\frac{12}{17}$ maketh $\frac{22}{17}$, but if the Denominators be not like, then you must make them like by the seventh Rule before taught.

What if there be three fractions of divers denominations to be added together, as $\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{5}$.

Then having reduced the two first to one selfe-denomination, adde the two Numerators together, and write under the summe thereof the common Denominator, that done, deale with that fraction last found, and with the third as you did before with the two fractions that had divers denominations and so you shall find that $\frac{2}{3}$ and $\frac{3}{4}$ being first brought to one denomination, and then added together do make $\frac{17}{12}$ whereto if you will adde $\frac{4}{5}$ then you must bring them againe to one selfe-denomination, and so you shall find that $\frac{17}{12}$ and $\frac{4}{5}$ do make $\frac{85}{60}$ and $\frac{48}{60}$ which last two Numerators being added together do make in all $\frac{133}{60}$ which is two Integrums and $\frac{13}{60}$, but a more ready way is to multiply all the Denominators together into one totall or common Denominator, and then for the severall Numerators multiply each

Numerator,

Numerator by all the Denominators, saving that which is under it selfe, and so is the former question performed at one worke or any such like how many soever.

Subtraction of Fractions.

HOw are Fractions to be subtracted one out of another, or out of *Integrums*?

First, make the Denominators like as you did before in Addition, then take the lesser Numerator out of the greater, and under the remainder thereof write the common Denominator, so shall you find that $\frac{3}{7}$ being taken out of $\frac{6}{7}$ there remaineth $\frac{3}{7}$, but if you would subtract fractions out of Integrums, then you must take one of the Integrums and breake it into parts. As for example, to subtract $\frac{1}{7}$ out of 9 Integrums, take one from 9, and breake it into parts making it $\frac{7}{7}$ for that is one Integrum, for whensoever the Numerator is made like and equall to the Denominator, it signifieth one Integrum, then take $\frac{3}{7}$ out of $\frac{7}{7}$ and there remaineth $\frac{4}{7}$ and 8 Integrums.

What is to be observed in breaking the *Integrum*?

In breaking the Integrum, you are to be directed alwayes by the Denominator of the fraction which you have to subtract, for the Integrum which should be the Numerator, must be equall in value to the said Denominator, as before is said.

Multiplication of Fractions.

HOw are Fractions to be multiplyed by Fractions, or *Integrums* by Fractions.

As touching fractions, multiply the Numerator one into another, and the Product thereof shall bee the Numerator, then multiply the Denominators in like manner, and the Product thereof shall be the Denominator, so shall you find that $\frac{5}{7}$ being multiplyed by $\frac{3}{4}$ do make $\frac{15}{28}$. But if you would multiply Integrums by fractions, then multiply the Integrums by the Numerator of the fraction, and under the Product thereof set downe the Denominator of the same fraction, drawing a line betwixt them, so shal yo find that 20 Integrums being multiplyed by $\frac{5}{12}$ do make $\frac{100}{12}$, that is to say eight Integrums and $\frac{4}{12}$ which is $\frac{1}{3}$, &c.

Divi-

Division of Fractions.

HOw are Fractions to be divided by Fractions, or *Integrums* by Fractions, or Fractions by *Integrums*?

The division of fractions is done by Multiplication thus, first set downe your Dividend on the left hand, and the Divisor alwayes on the right hand, and then draw two crosse lines like a Saint Andrewes Crosse betwixt them which shall direct you in your working: As for example, if you would divide $\frac{3}{2}$ by $\frac{4}{5}$ set them downe thus;

Dividend. Divisor.

$\frac{3}{2} \times \frac{4}{5}$ and worke as followeth: first multiply the Numerator of the Dividend, by the Denominator of the Divisor, and the Product thereof shall be the Numerator, then multiply the Denominator of the Dividend by the Numerator of the Divisor, and that shall be the Denominator, so shall you find that $\frac{3}{2}$ being divided by $\frac{4}{5}$ the Quotient is $\frac{15}{8}$ or $1\frac{7}{8}$, but if you would divide Integrums by fractions, or contrariwise fractions by Integrums, then make of Integrums a fraction, by setting downe 1 in the place of a Denominator under the Integrums, and worke as before, so shall you find that $\frac{7}{4}$ Integrums being divided by $\frac{1}{4}$ do make $\frac{28}{4}$, contrarily if you divide $\frac{1}{4}$ by $\frac{7}{4}$ Integrums, there will arise $\frac{4}{28}$.

What if I have to divide Fractions annexed to *Integrums*?

Then you must first reduce each Integrum with his fraction annexed, into one selfe-fraction by the fourth Rule of fractions before taught. As for example, if you would divide nine Integrums having $\frac{1}{4}$ thereunto annexed, by three Integrums and $\frac{1}{3}$, here the fourth Rule of fractions teacheth you first to multiply the Integrum 9 by 4, which is the Denominator of the fraction annexed, the Product whereof is 36, whereunto by adding the Numerator of the same fraction which is 3, you shal make the Numerator of the Dividend to be 39, under which you must set the Denominator 4, thus $\frac{39}{4}$, and that is your whole Dividend, then having in like maner brought the divisor which

is

is 3 Integrums and $\frac{1}{3}$ into one ſelfe-fraction, worke as the former Rule of diviſion teacheth you, and you ſhall produce $2\frac{17}{51}$ which is two Integrums and $\frac{17}{51}$.

Chap. IX.

The Rule of Three belonging to Fractions.

WHat order is to bee obſerved in this Rule having to deale with Fractions only?

The ſelfe-ſame order that hath bæne taught before touching Integrums: for in working with fractions you muſt have alſo three ſeverall numbers, and you muſt ſæ that the firſt and third numbers bee of one ſelfe-denomination, and that number is to be placed alwayes in the third place whereunto the queſtion is annexed, and then to multiply the ſecond by the third, and to divide by the firſt, and ſo the fourth number which you ſæke to know, ſhall appeare: As for example, if $\frac{1}{4}$ of an ell of fine Holland coſt mæ $\frac{3}{5}$ of an Engliſh Crowne, in value 15 groats, what ſhall $\frac{1}{6}$ of an ell coſt me? here firſt you muſt ſet downe your three ſeverall numbers in order thus: $\frac{1}{4}$ ell $\frac{3}{5}$. $\frac{1}{6}$ ell. as the firſt and third may be one ſelfe-denomination: then multiply the ſecond and third fraction the one into the other, which will make $\frac{3}{30}$ and that being divided by $\frac{1}{4}$ which is in the firſt fraction, will produce $\frac{12}{30}$ of a Crowne. Or rather by this briefe Rule multiply the Numerator of the firſt number given, by the Denominator of the ſecond, and that Product by the denominator of the third number given, this Product is Denominator or Diviſor. Then multiply by the Numerator of the ſecond number given by the Numerator of the third, and the Product by the Denominator of the firſt number given, this Product is Numerator or Dividend. The value of which fraction if you ſæke to know by helpe of the firſt Rule of fractions, teaching you to multiply the Numerator of the fraction, by the knowne parts of a Crowne, which are five ſhillings or fifteene groats, you ſhall find the value of that fraction to bee eleven groats and $\frac{5}{12}$ of a groat,

which is one Farthing and somewhat more, supposing the least knowne parts of the Groat to be sixteene Farthings.

Chap. X.

The golden Rule reverse, called in Latine Regula reversa, *that is to say, turned backward.*

WHat is the order of this Rule?

Multiply the first by the second, and divide the Product thereof by the third. Or multiply the numerator of the first by the numerator of the second, and that Product by the denominator of the third number given, this last Product is numerator or dividend. Then multiply the numerator of the third by the denominator of the second, and that Product by the denominator of the first number given: this Product is denominator or divisor, as if a penny-loafe must weigh two pound, Wheat being at three shilling a bushell, what shall a penny-loafe weigh when Wheat is at two shillings the bushell? the question must be framed thus, if three shillings require two pound weight, what shall two shillings have? then by working according to this Rule, you shall find that the penny-loafe must weigh three pound.

Another example of the same Rule.

I would know how many yards of Bayes bearing in breadth $\frac{7}{4}$ will suffice to line seven yards of silke bearing in breadth three quarters and a halfe. Here you must frame your question thus, if $\frac{3}{4}$ and $\frac{1}{2}$ require seven yards, what shal $\frac{7}{4}$ require? but because the first and third number of this question are not of one selfe-denomination by reason of the fraction annexed to the first number, you shall do well to reduce the first and third number both into halfe quarters, and then to worke as though they were all integrums, which is more easie then to make all the numbers fractions; wherefore say thus: If seven halfe quarters do require seven yards, what shall fourteene halfe quarters require? and by working by the Rule Reverse, you shall find in the Quotient three yards of Bayes and a halfe.

Chap.

Chap. XI.

The double Rule called in Latine Regula duplex.

WHereto serves this Rule? and what order is to be observed therein?

This Rule serveth to unfold two Questions wrapt in one, as thus. If I pay 4 d for the carriage of 20 l weight 30 miles, what shall I pay for the carriage of 50 l weight 60 miles? Here of this and such like demands, you must make two sundry Questions, and the fourth summe of the first Question being found, shall bee the second or middle number of the second Question, wherefore frame your first Question thus, if 20 l cost 4 d, what shall 50 l cost? and you shall find that it will cost you 10 d, then say if 30 miles cost 10 d, what shall 60 miles cost? and you shall find that it will cost 20 d. And note that each of these two Questions is to bee wrought by the common Rule of Three, that is to say, by multiplying the second into the third, and by dividing the Product thereof by the first, and the fourth found number of the first Question must bee the second, or middle number of the second Question, as in the former example, you see that 10 d, which was the fourth found number, is here the middle number of the second Question.

Another Example.

IF 25 pound do gaine me eight pound in foure yeares, how much shall 100 pound winne one in ten yeares? both these Questions are also to be wrought by the common Rule of three. Wherefore set downe the first Question thus: if 25 pound yeldeth eight pound, what shall 100 yeld? and you shall find 32 pound: then say if foure yeares yeldeth 32 pound, what shall ten yeares yeld? and you shall find eighty pound. But note that these double Questions may bee put in such sort, as you must worke the first or second Question sometimes by the Rule Re-

verse.

verse. As in this Question here following, if 6 l winne eight Crownes in ten yæres, in how many yæres shall 3 l winne 12 Crownes? here frame your first Question thus: if 6 l require ten yæres, how many yæres shall 3 l require? and in working this Question by the Rule Reverse, you shall find twenty yæres, then for the second Question say thus, if eight Crownes require twenty yæres, how many yæres shall 12 Crownes require? here if you worke by the common or direct Rule of Thræe, you shall find thirty yæres.

Another example.

IF seven horses do eat 12 bushels of Oats in twenty dayes, how many bushels shall fourtæne horses eat in fiftæne dayes? here frame your first Question thus, if seven horses eat 12 bushels, what will fourtæne horses eat? and in working by the common or direct Rule of Thræe, you shall find in the Quotient 24 bushels, then frame your Question thus, if 20 dayes require 24 bushels, what shall fiftæne dayes require? here in working by the common or direct Rule of thræe, you shall find in the Quotient eightæne bushels.

Another Example.

IF ten Reapers reape fiftæne Acres in seven dayes, in how many dayes shall sirtæne Reapers reape twenty Acres? here frame your first Question thus, if ten Reapers require seven dayes, how many dayes shall sirtæne Reapers require? which Question must be wrought by the Rule Reverse, and so you shal find foure dayes and $\frac{1}{2}$ of a day which is nine houres, then say, if fiftæne Acres require foure dayes and $\frac{1}{2}$ of a day, how many dayes shal twenty Acres require? and in working this second Question by the common or direct Rule of thræe, belonging to fractions, as is before taught, you shall find that twentie Acres will require five dayes and $\frac{10}{11}$ of a day, which is twenty houres. But it were more ease in this second Question to reduce the dayes into houres, by multiplying the foure dayes by 24 houres, the Product whereof will be 96 houres, whereunto if you adde the odde nine houres, it will make in all 105 houres,

houres, which being multiplyed by the third number of this second question, which is 20, the product shall be 2100 houres, which divided by the first number of the said Question which is 15, you shall find in the Quotient 140 houres, which if you divide againe by 24, you shall find in the Quotient 5 dayes, and the remainder to be 20 houres, which agreeth in all points with the first manner of working by fractions, and is the easier way of the two.

How shall I know, having to worke by this double Rule, when to use the Rule Reverse.

By considering whether the third number requireth more or lesse of time, or of any other measure or quantity, as in the former Example of Wayes for Lining, the more breadth it had, the lesse did serve for lining. Againe, in the Example of gaine by yeares, of 6 £, and 3 £, you did see that 3 £ require more yeeres then 6 £, and therefore that first Question was wrought by the Rule Reverse: Also in this last Example of the Reapers, the more Reapers the lesse time they require, and therefore that question was wrought by the Rule Reverse.

Chap. XII.

The Rule of Fellowship.

What doth this Rule teach?

To know the gaine or losse of such as do make a stocke, and do occupie together in the Trade of Merchandize: As for Example, foure Merchants did put their money in lot in this manner. The first brought thirty Crownes, the second fifty, the third forty, and the fourth an hundred, and with these portions they gained 3000 Crownes, the question is how much every one shall have to his share of that gaine according to the portion which he brought. To know this, you must first gather all the severall portions together by Addition into one Summe, and you shall find the Summe of the portions to bee 240

Crownes,

Crownes, then say if 240 Crownes do gaine 3000 Crownes, what shall 30 gaine, and after this manner worke by the common or direct Rule of three, with all the rest of the portions, and so you shall find that he which brought 30 ought to have 375 Crownes, and he that brought 50 ought to have 625 Crownes, and hee that brought 60 ought to have 750 Crownes, and hee that brought an hundred ought to have 1250, which maketh in all 3000 Crownes, for that is the very summe of the gaine before set downe, the order of working whereof, this figure plainly sheweth.

240 —— 3000 —— { 30 — 375. / 50 — 625. / 60 — 750. / 100 — 1250.

The common Divisor which is the sum of the particular portions added together — { the generall gaine. / the particular portiõs. / every mans singleshare }

A like Example of losse received by Shipwrack.

THree Merchants do venture their goods in one ship, the goods of the first were worth 300 Crownes, the second 400, the third 500, there were as much goods cast out as was worth 100 Crownes, the Question is how much every one should lose according to his portion, here worke as before, and you shall find that every one shall lose so much as this Figure following sheweth.

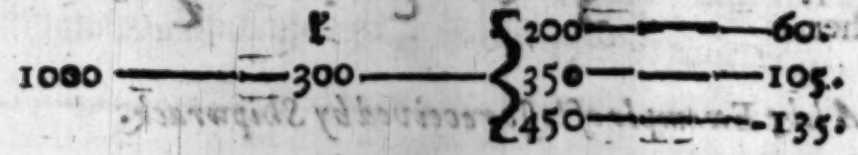
1200 —— 100 —— { 300 — 25. which is five groats. / 400 — 33. 1/3 / 500 — 41. 2/3 which is ten groats.

The common Divisor which is the sum of the particular portions added together — { the generall losse. / the particular portions / every mans severall losse. }

Here to know whether the three ſeverall loſſes do make up the generall loſſe, do thus, firſt adde the Integrums of the ſeveral loſſes together, the ſum whereof will be 99, whereunto if you adde the two fractions which do make one whole Crowne, the ſum will be 100, a number like unto the generall loſſe.

Another Example.

THree Merchants have bought 1000 l. of Pepper for three hundred Crownes, the firſt taketh 200 l, the ſecond 350 l, and the third 450 l, what ſhall every man pay according to the portion which he hath received, then ſay if 1000 l be worth 300 Crownes, what is 200 l worth: here working by the common or direct Rule of Three with every mans ſeverall portion received, you ſhall find that the firſt muſt pay 600 Crownes, the ſecond 105 and the third 135 Crownes, as this figure here following ſheweth.



The common Diviſor. } the generall price. } the ſeveral portions received. } every mans particular payment. }

Another like Example of diver's diſtance of time.

THree Merchants occupying together did gaine 2345 Crownes, the firſt put in forty Crownes for the ſpace of fourteene months, the ſecond put in fifty Crownes, for the ſpace of eight months, and the third put in 85 Crownes for the ſpace of ſix months: the queſtion is, how much every one ſhall have after the rate of his money, and according to the quantity of time: This is to be wrought according to the Rule of fellowſhip thus, multiply every mans money by his time, eyther of which, that is to ſay, the money and the time muſt be of one ſelfe Denomination, then adde the Summes of thoſe ſeverall portions

together

together, and the totall summe of such Addition shall be the first number, and the common gaine shall be the second number, and the third number shall be every mans money multiplyed by his time, then in working by the common or direct Rule of thrée, you shall find that every man shall have such share as this figure here following sheweth.

1470	2345	560	893. $\frac{1}{3}$
		400	638. $\frac{2}{21}$
		510	813. $\frac{4}{7}$

The common Divisor which is the totall summe of the severall portions of mony and time added together.	the generall summe of the gaine.	the particular portions of the money multiplied by his time.	every mans single share.

Now if you would know whether the single shares in this example do make up the generall summe of the gaine, then adde together the Integrums or whole summes of every mans single share, and you shall find the summe to be but 2344, which wanteth one whole Integrum of the second or middle number of the question, which you shall easily supply by adding the thrée severall fractions together according to the Rule of Addition of fractions before taught, for so shall you find the said thrée fractions to make in all one whole Integrum, which being added to 2344, will be answerable to the second number of the Question, which as you sée in the Example, is 2345. And remember in adding together the said thrée fractions, to set them in this order $\frac{1}{3}|\frac{2}{21}|\frac{4}{7}$, and then to bring them to one selfe-denomination, as the seventh Rule of fractions teacheth you. But first because you shall find the thrée remainders after the first thrée Divisions of the thrée severall portions made by the first common Divisor of the Question to be written in many figures, you must set them downe in fewer figures, according as the fifth Rule of fractions teacheth you, so shall you find the first remainder contayning $\frac{490}{1470}$ to be no more but $\frac{1}{3}$, and the second remainder contayning $\frac{140}{1470}$ to bee $\frac{2}{21}$, and the third remaynder contayning $\frac{840}{1470}$ to be $\frac{4}{7}$, which thrée fractions

being added together according to the seventh Rule of fractions, which will $\frac{441}{441}$ which is one whole Integrum, and must bee added by the name of 1, to the middle or second number of the question as I have said before: you may also make up the foresaid number by seeking to know the value of every fraction annexed to the Integrums, according as the first Rule of fractions teacheth, for so shall you find the value of the first fraction to be five groats, and the value of the second fraction to bee one groat six farthings, and $\frac{21}{147}$ of a farthing, and you shall find the value of the third fraction to be eight groats, nine farthings, and $\frac{1}{7}$ of a farthing, and if you adde $\frac{21}{147}$ and $\frac{1}{7}$ of a farthing together, you shall find that it will make $\frac{147}{147}$ which is one whole Integrum, or one whole farthing. Now, if you adde six, nine and one farthings together, it will make in all sixteene farthings, and that is one groat, which being added to the fourteene groats before found out, will make in all fifteene groats, which is one Crowne, and that being added to the summe 2344 will make it 2345 Crownes, which is a number agreeable to the middle summe of the question. Truly if you exercise your selfe in this and such like questions, it will make you perfect not only in Addition, Subtraction, Multiplication, and Division of whole numbers, but also of fractions, and almost in all the other Rules belonging as well to fractions, as to Integrums. Wherefore I would wish you often to use your Pen therein.

Having hitherto treated of the foure speciall kinds of Arithmeticke, that is, of Addition, Subtraction, Multiplication, and Division, as well belonging to whole numbers, as to fractions: and also shewed the use of the Golden Rule, otherwise called the Rule of Three, and of all the three kinds therof, that is, the common Rule, the Rule Reverse, and the Double Rule, and given Examples, how and when every one is to be used, together with the Rule of Fellowship, necessary for them that use any trade of Merchandize, I thinke good now to speake somewhat of Arithmeticall and Geometricall progression, and also of proportion, and of the three kinds thereof, that is, Arithmeticall, Geometricall, and Musicall proportion, and then of the extraction of Roots both square and cubicall, and last of all, of

Astro-

Astronomicall fractions, shewing how they are to be added, subtracted, multiplyed, and divided.

CHAP. XIII.

Of Progression, and how manifold it is.

WHat is Progression?

It is a certaine order of proceeding with divers Numbers, in such sort as every one may exceed each other, either by like difference of quantity, or else by likenesse of Proportion, whereof springeth two kinds of Progression, the one called Arithmeticall, and the other Geometricall.

What is Arithmeticall Progression?

It is that which proceedeth by like difference of quantity, as thus; 3,5,7,9,11,13, whereof every one exceedeth his fellow by the difference of two.

What is Progression Geometricall?

It is that wherein every number exceedeth his fellow be like Proportion, for as six containeth three twice, so doth twelve containe six twice, &c.

CHAP. XIIII.

Of Addition belonging to Progression Arithmeticall.

HOw are numbers being set in order according to Progression Arithmeticall, to be added together, and to be reduced into one totall summe?

Thus, first see how many severall numbers there be in all, and note the summe by it selfe, then adde the first number of the Progression to the last thereof, and note that sum by it selfe, that done, multiply one of these two reserved sums by the one halfe of the other, and

you shall have the totall summe. As for Example, let these be your Progression, 6, 10, 14|18|22|26|30|34| wherein every number exceedeth his fellow by the difference of foure: here having told the numbers, I find them to be eight, which I set down by it selfe, then I adde 6, which is the first number, to 34, which is the last, and the summe thereof is 40, which I also set downe by it selfe, then in multiplying 40 by 4, which is the one halfe of 8, before set downe a part by it selfe, I find the totall summe of the Progression to be 160.

Chap: XV.

Of Addition belonging to Progression Geometricall.

How are numbers being set in order according to Progression Geometricall to be added and to be brought into one summe?

Thus; multiply the last summe of the Progression by the number of the Proportion, wherby such Progression proceedeth, and from the Product of that multiplycation, subtract the first number of the Progression: that done, divide that which remayneth by a number which is lesse than the Proportion by one, and the Quotient of such division shall shew you the totall summe of the said Progression. As for Example, let this be the Progression, 2, 6, 18, 54, 162, 486, 1458, the Proportion of which Progression is triple, wherefore according to the Rule, I multiply the last number of the Progression by 3, and the Product of such multiplication amounteth to 4374. out of which summe I subtract the first number which is 2, and there remayneth 4372, which I divide by 2, (for it is lesse than the Proportion 3 by 1,) and so I find in the Quotient 2186, which is the totall summe of the Proportion Geometricall.

Is there no briefer way of adding such kinds of Progression?

Yes indeed, but not so plaine as this way is, and therefore I thinke not good to trouble your memory therewith.

Chap. XVI.

Of Proportion.

WHat is Proportion?

Proportion is taken generally for the comparing of two divers quantities or numbers together, shewing what likenesse is betwixt them. But before we deale with Proportion, and with the three kinds thereof, that is, Proportion Arithmeticall, Geometricall, and Musicall: You have first to understand, that of numbers some are said to be abstract, and some concrete.

The Abstract are such as are not tyed to any Denomination, and such as are two-fold, that is, Absolute and Relative.

The Absolute are simply pronounced without having any relation to any other number, measure, or quantity, as 2, 3, 4, &c. and all numbers whatsoever, that are without Denomination, and are not attributed to any other thing.

The Relative are those which have relation one to another, which may be three manner of wayes. First in respect of difference which is found by subtraction: secondly, in respect of the Quotient found by division: thirdly, in respect of both. Of the first way springeth Arithmeticall Proportion: Of the second way Geometricall Proportion: And of the third Musicall Proportion.

Chap. XVII.

WHat is Arithmeticall Proportion?

Arithmeticall Proportion, unproperly so called, (because it is no Proportion indeed) is when many severall numbers have one selfe and like difference, as 8, 12, 16, which do only differ one from another by 4. and this Proportion is two-fold, that is, Continuall and Disiunct.

Continuall is, when any numbers proceed with like difference, as hath beene said before when we spake of Arithmeticall Progression, as 8, 12, 16, 20, &c. whose difference betwixt every two numbers is foure.

The Disiunct, is when many numbers being severally propounded, the difference of the first two numbers is not like to the difference that is betwixt the second and the third, and so forth, as 5, 8, 4, 7, for 8 differeth from 5 by 3, and 4 from 8, by 4, and 7 from 4 by 3. Now of the second way of comparing, which is done by division, springeth as hath beene said before, Geometricall proportion.

Chap. XVIII.

What is Geometricall Proportion?

Geometricall Proportion, is that which sheweth what part or parts one number is of another number, as three is the halfe of six, which proportion is found by division, wherein you have to note, that if the Diviſor be greater than the Dividend, then it is to be made a fraction, as in the former example: if you would divide three by six, then you must make it a fraction thus, $\frac{1}{2}$ or $\frac{3}{6}$, and this kind of Proportion, which may bee truly and properly called a proportion indeed, or rather a proportionality, is said to be two-fold, that is Direct, and Reverse, and the Direct is either Coniunct, or Disiunct.

Coniunct differeth not from Geometricall Progression, before taught.

Disiunct proportion Geometricall, consisting most commonly of foure numbers, or of three at the least, is when there is not like proportion betwixt the second and the third, that is, betwixt the first and the second, or betwixt the third and the fourth, as 3, 6, 4, 8, for here six contayneth foure once, and one halfe thereof, which is called Proportio Sesquialtera, and six contayneth three twice, which is called Proportio dupla: and so is eight to foure.

Chap. XIX.

AGaine, proportion Reverse differeth not from the Rule of Three, called Regula reversa. But you have to understand, that the two chiefe and speciall kinds of Geometricall proportion, are these, that is, proportion of equality, and proportion of inequality.

Proportion of equality, is when two numbers compared together, are equall the one to the other, as 3 to 3, 4 to 4.

The proportion of inequality, is when two unequall numbers are compared together, as 6 to 5, 4 to 9, and of this there are two kinds, that is, proportion of the greater inequality, and proportion of the lesser inequality.

Chap. XX.

PRoportion of the greater inequality, is when the greater number is compared to the lesser number, as six to five.

Proportion of the lesser inequality, is when the lesser is compared to the greater, as five to six.

Of proportion of the greater inequality there be two kinds, Simplex *and* Multiplex, *that is to say, simple and manifold.*

SImplex, is when the Antecedent, that is to say, the former number containeth the Consequent, that is to say, the latter number once and somewhat more, which overplus must alwayes be lesse then the Consequent it selfe, as five containeth foure once and one part thereof more, for if you divide five by foure, the Quotient will be 1, and $\frac{1}{4}$ over. Againe, this proportion is two-fold, that is, Superparticular, and Superpartient,

Super-

Superparticular, is when the Antecedent contayneth the Consequent once and some one part thereof, as 3 to 2, for 3 contayneth 2 once and one halfe thereof, which is called Sesquialtera, or as 4 to 3, for 4 contayneth 3 once and one third part thereof and is called Sesquitertia. The like is to be said of Sesquiquarta, Sesquiquinta, Sesquisexta, and so forth infinitely, as this Table sheweth.

Superparticular proportions are these & such like.

Sesquialtera,	as 3 to 2, 6 to 4, 9 to 6,	which is as much as	$1\frac{1}{2}$
Sesquitertia,	as 4 to 3, 8 to 6, 12 to 9,		$1\frac{1}{3}$
Sesquiquarta,	as 5 to 4, 10 to 8, 15 to 12,		$1\frac{1}{4}$
Sesquiquinta,	as 6 to 5, 12 to 10, 18 to 15,		$1\frac{1}{5}$
Sesquisexta,	as 7 to 8, 14 to 12, 21 to 18,		$1\frac{1}{6}$
Sesquiseptima,	as 8 to 7, 16 to 14, 24 to 21,		$1\frac{1}{7}$
Sesquioctava,	as 9 to 8, 18 to 16, 27 to 24,		$1\frac{1}{8}$
Sesquinona,	as 10 to 9, 20 to 18, 39 to 27,		$1\frac{1}{9}$
Sesquidecima.	as 11 to 10, 22 to 20, 33 to 30		$1\frac{1}{10}$
Sesquiundeci.	as 12 to 11, 24 to 22, 36 to 33		$1\frac{1}{11}$
Sesquiduode.	as 13 to 12, 26 to 24, 39 to 36		$1\frac{1}{12}$

But Superpartient is when the Antecedent contayneth the Consequent once and some parts thereof, that is to say, more parts then one, as five to three, for five contayneth three once and two third parts thereof, which is called Superbipartiens tertias, of which kinds are these set downe in the Table following.

Tertias,

Proportions superpartient are these, and such like.

Superbipartiens.	Tertias, as 5 to 3, 10 to 6, 15 to 9,	which is as much as	$1\frac{2}{3}$
	Quintas, as 7 to 5, 14 to 10, 21 to 15,		$1\frac{2}{5}$
	Septimas, as 9 to 7, 18 to 14, 27 to 21,		$1\frac{2}{7}$
	Nonas, as 11 to 9, 22 to 18, 33 to 27,		$1\frac{2}{9}$
	Vndecimas, as 13 to 11, 26 to 22, 39 to 33.		$1\frac{2}{11}$
	Decim. tertias, as 15 to 13, 30 to 26, 45 to 39,		$1\frac{2}{13}$
Supertripartiens.	Quartas, as 7 to 4, 14 to 8, 21 to 12.	which is as much as	$1\frac{3}{4}$
	Quintas, as 8 to 5, 16 to 10, 24 to 15,		$1\frac{3}{5}$
	Septimas, as 10 to 7, 20 to 14, 30 to 21,		$1\frac{3}{7}$
	Octavas, as 11 to 8, 22 to 6, 33 to 24.		$1\frac{3}{8}$
	Decimas, as 13 to 10, 26 to 20, 39 to 30		$1\frac{3}{10}$
	Vndecimas, as 14 to 11, 28 to 22, 24 to 33,		$1\frac{3}{11}$
Superquadripartiens.	Quintas, as 9 to 5, 18 to 10, 27 to 15,	which is as much as	$1\frac{4}{5}$
	Septimas, as 11 to 7, 22 to 14, 33 to 21,		$1\frac{4}{7}$
	Nonas, as 13 to 9, 26 to 18, 39 to 27,		$1\frac{4}{9}$
	Vedecimas, as 15, to 11, 30 to 22, 45 to 33,		$1\frac{4}{11}$
	Decim. tertias, as 17 to 13, 34 to 26, 51 to 39,		$1\frac{4}{13}$
	Decim. quintas, as 19 to 15, 38 to 30, 57 to 45		$1\frac{4}{15}$
Superquintupartiens.	Sextas, as 11 to 6, 22 to 12, 33 to 18,	which is as much as	$1\frac{5}{6}$
	Septimas, as 12 to 7, 24 to 14, 36 to 21,		$1\frac{5}{7}$
	Octavas, as 13 to 8, 26 to 16, 39 to 24.		$1\frac{5}{8}$
	Nonas, as 14 to 9, 28 to 18, 42 to 27,		$1\frac{5}{9}$
	Vndecimas, as 16 to 11, 32 to 22, 48 to 33,		$1\frac{5}{11}$
	Duodecimas, as 17 to 12, 34 to 24, 51 to 36.		$1\frac{5}{12}$
Supersextupartiens.	Septimas, as 13 to 7, 26 to 14, 39 to 21,	which is as much as	$1\frac{6}{7}$
	Vndecimas, as 17 to 11, 34 to 22, 51 to 33,		$1\frac{6}{11}$
	Decim. tertias, as 19 to 13, 38 to 26, 57 to 39,		$1\frac{6}{13}$
	Decim. septim. as 23 to 17, 46 to 34, 60 to 51,		$1\frac{6}{17}$
	Decim. nonas, as 25 to 19, 50 to 38, 75 to 57,		$1\frac{6}{19}$
	Vicessimas tertias, as 29 to 23, 58 to 46,		$1\frac{6}{23}$

Hitherto of Simplex proportio. Now of Multiplex proportio.

Multiplex proportio is, when the Antecedent containeth the Consequent more then once, as 6 to 2, for 6 containeth 2 three times,

times which is called Tripla proportio. Also 12 to 5, for 12 comprehendeth 5 twice and $\frac{2}{5}$. And this Multiplex proportio is twofold that is, either exact or not exact.

Multiplex exact, is when the Antecedent containeth the Consequent more then once, and nothing remayneth, as 4 to 2, 6 to 3, &c. whereof are infinite kinds, as Dupla, Tripla, and so forth, as this Table sheweth.

The kinds of Multiplex exact, are these and such like.

Dupla, as 4 to 2, 6 to 3, 8 to 4,	which is as much as	$\frac{4}{2}$ $\frac{6}{3}$ $\frac{8}{4}$
Tripla, as 6 to 2, 9 to 3, 12 to 4,		$\frac{6}{2}$ $\frac{9}{3}$ $\frac{12}{4}$
Quadrupla, as 8 to 2, 12 to 3, 16 to 4		$\frac{8}{2}$ $\frac{12}{3}$ $\frac{16}{4}$
Quintupla, as 10 to 2, 15 to 3, 20 to 4,		$\frac{10}{2}$ $\frac{15}{3}$ $\frac{20}{4}$
Sextupla, as 12 to 2, 18 to 3, 24 to 4,		$\frac{12}{2}$ $\frac{18}{3}$ $\frac{24}{4}$
Septupla, as 14 to 2, 21 to 3, 56 to 8,		$\frac{14}{2}$ $\frac{21}{3}$ $\frac{56}{8}$
Octupla, as 16 to 2, 24 to 3, 32 to 4,		$\frac{16}{2}$ $\frac{24}{3}$ $\frac{32}{4}$
Nondupla as 18 to 2, 27 to 3, 36 to 4,		$\frac{18}{2}$ $\frac{27}{3}$ $\frac{36}{4}$
Decupla, as 20 to 2, 30 to 3, 40 to 4,		$\frac{20}{2}$ $\frac{30}{3}$ $\frac{40}{4}$
Undecupla, as 22 to 2, 33 to 3, 495 to 45,		$\frac{22}{2}$ $\frac{33}{3}$ $\frac{495}{45}$

But Multiplex not exact, is when the Antecedent containeth the Consequent more then once, and something remayneth over, as 5 to 2, for 5 containeth 2 twice, and one remayneth: and this is also two-fold, that is, Multiplex superparticularis, and Multiplex superpartiens.

Multiplex superparticularis, is when the Antecedent containeth the Consequent more then once, and one only remayneth, as 7 to 3, for 7 containeth 3 twice, and one only remayneth, whereof are divers kinds, as Duplex sesquialtera, Duplex sesquitertia, Triplex sesquisexta, and so forth, as the Table hereafter following sheweth.

But Multiplex superpartiens, is when the Antecedent containeth the Consequent more then once, and the remainder is more then 1, as 8 to 3, for 8 containeth 3 twice, and 2 thirds over, whereof there be many kinds, as Dupla superbipartiens tertias, Dupla supertripartiens quartas, and so forth, as this Table following sheweth; which comprehendeth both kinds, that is, Multiplex superparticularis, and Multiplex superpartiens.

Multiplex

Multiplex not exact, is either

Multiplex Superparticularis or else

Duplex
- Sesquialtera, as 5 to 2, 15 to 6,
- Sesquitertia, as 7 to 3, 21 to 9,
- Sesquiquinta, as 11 to 5, 22 to 10.

Triplex
- Sesquisexta, as 19 to 6, 38 to 12,
- Sesquiseptima, as 22 to 7, 44 to 14,
- Sesquioctava, as 25 to 8, 50 to 16,

Quadruplex
- Sesquinona, as 37 to 9, 148 to 36,
- Sesquidecima, as 41 to 10, 82 to 20,
- Sesquiundecima, as 45 to 11, 90 to 22

which is $2\frac{1}{2}$, $2\frac{1}{3}$, $2\frac{1}{4}$; $3\frac{1}{6}$, $3\frac{1}{7}$, $3\frac{1}{8}$; $4\frac{1}{9}$, $4\frac{1}{10}$, $4\frac{1}{11}$

Multiplex not exact, is either

Multiplex Superparticularis or else

Dupla.
- Superbipartiens: Tertias, as 8 to 3, Quintas, as 21 to 5,
- Supertripartiens: Quartas, as 11 to 4, Quintas, as 13 to 5,
- Superquadripartiens: Quintas, as 14 to 5, Septimas, as 18 to 7,

Tripla.
- Superquintupartiens: Octavas, as 29 to 8, Nonas, as 34 to 9,
- Supersextupartiens: Undecimas, as 39 to 11, Decimtertias, as 45 to 13,
- Superseptupartiens: Octavas, as 32 to 8, Nonas, as 34 to 9,

Quadrupla.
- Superbipartiens: Tertias, as 14 to 3, Quintas, as 22 to 5,
- Superquadripartiens: Quintas, as 24 to 5, Nonas, as 40 to 9,
- Superquintupartiens: Sextas, as 29 to 6, Septimas, as 33 to 7,

which is $2\frac{2}{3}$, $2\frac{2}{5}$; $2\frac{4}{5}$, $2\frac{4}{7}$; $3\frac{1}{8}$, $3\frac{1}{9}$; $3\frac{6}{11}$, $3\frac{6}{13}$; $3\frac{7}{8}$, $3\frac{7}{9}$; $4\frac{2}{3}$, $4\frac{2}{5}$; $4\frac{4}{5}$, $4\frac{4}{9}$; $4\frac{5}{6}$, $4\frac{5}{7}$

Thus

Thus much of Proportion of the greater inequality: Now we will speak some what of Proportion of the lesser inequality.

Chap XXI.

PRoportion of the lesser inequality, is when the Antecedent is lesse than the Consequent, as 2 to 3, 5 to 7, for if you divide 2 by 3, the Quotient is $\frac{2}{3}$, if 5 by 7, then the Quotient is $\frac{5}{7}$, and this Proportion hath the same names which the proportion of the greater inequality hath, saving that you must adde to the beginning of every name this word Sub, as Subdupla, Subtripla, Subsesquialtera, &c. as 1 to 2, is Subdupla Proportio, 3 to 9, is Subtripla Proportio, &c.

Chap. XXII.

Of Musicall Proportion, called in Latine Harmoniaca Proportio.

MUsicall Proportion, which requireth three numbers at the least, is when the first number hath the same Proportion unto the third, which the difference betwixt the first and the second hath to the difference which is betwixt the second and the third, as 3, 4, and 6, for looke what Proportion 3 hath to 6, which is Subdupla, the same hath the difference betwixt 3 and 4 which is 1, to the difference betwixt 4 and 6 which is 2, for 1 to 2 is Subdupla, and this is called Musicall Proportion, because the numbers therein have the same Proportions one to another, which are found to bee in Musicall Consorts, as 6, 4 and 3, for the Proportion which 6 hath to 4, is Sesquialtera, called of the Musitians Diapente, or a fifth, and the Proportion which 4 hath to 3, is Sesquitertia, called

of

of the Musitians Diatesseron, or a fourth, and the Proportion which 6 hath to 3, is Dupla, called a Diapason, or an eight, and of this Musicall Proportion there be two kinds, that is, Simple and Compound.

It is said to be Simple when it consisteth only of three numbers.

And it is called Compound when it consisteth of more then three numbers: And of Compound there bee two kinds, that is, unproper and proper.

The unproper is, when to 3 numbers given, 2 other severall numbers are ioyned, which do containe the same Proportions with the third, which the first 3 numbers have one to another: As for example, let begiven these 3 numbers 3, 4, 6, unto which if you ioyne 8 and 12, here 6, 8, 12 have the same proportion one to another which 3, 4 and 6 have amongst themselves, for like as here betwixt 6 and 8 the proportion is Subsesquitertia, and betwixt 6 and 12 it is Subdupla, and betwixt 8 and 12 it is Subsesquialtera, so betwixt 3 and 4 is Subsesquitertia, and betwixt 3 and 6 is Subdupla, and betwixt 4 and 6 is Subsesquialtera.

Compound proper is, when divers numbers in Musicall Proportion standing together, and the first being omitted, the 3 next do continue still in Musicall Proportion. Also when omitting the 2 first numbers, the 3 next following are in Musicall Proportion. And when omitting the first 3, the next 3 are in Musicall Proportion, and so forth, how many numbers soever there bee: As for example, 10, 12, 15, 20, 30, these are numbers belonging to the proper kind of Compound Musicall Proportion, for 10, 12, and 15, are in Musicall Proportion, then omitting 10, which is the first number, 12, 15 and 20 are still in Musicall Proportion, and if you omit the 2 first numbers, that is, 10 and 12, then the next 3, that is 15, 20, and 30, are still in Musicall Proportion, and so forth, how many soever there be of them, so as they be in Musicall Proportion: But our Musitians do make no more but 8 Musicall Proportions in all, that is,

Dupla.

Dupla.	which are thus named.	Diapason.
Tripla.		Diapason diapente.
Quadrupla.		Bis diapason.
Sesquialtera.		Diapente.
Sesquitertia.		Diatesseron.
Sesquiquarta.		Diatonus semitonus.
Dupla superbipartiens.		Diapason diatesseron.
Sesquioctava.		Tonus.

The use whereof is to bee learned at the hands of the Musitians.

CHAP. XXIII.

But now because it is not enough to know the foresaid proportions and their names, unlesse you can both adde, subtract, multiply, and divide them when neede is, I will therefore briefly here set downe the order thereof, and first shew how to set them downe in writing, and then how they are to be added, subtracted, multiplyed, and divided.

They are to bee set downe in writing, in like manner that Fractions are wont to be set downe, saving that as in Fractions the upper number is called Numerator, and the inferior number Denominator, so in proportions the upper number is called Antecedent, and the inferior number Consequent, but whereas in Fractions, there is wont to bee drawne a little line betwirt the Numerator and the Denominator, in proportions no line is drawne betwirt the Antecedent and the Consequent, but the Antecedent is set over the Consequent, without any line drawne betwirt them thus.

And looke as Fractions are to be written in as few Figures as may be, so are proportions to be set downe in 3.|4.|2.| so smal numbers as may bee, which is to bee done by the 5.|7.|1.| selfe same Rule that Fractions are.

Againe, the order of adding, subtracting, multiplying, and dividing

dividing of Proportions is much like to that of Fractions, saving that the adding of Proportion is to bee done in such manner as Fractions are to be multiplyed, and the subtracting of Proportions is like to the Division of Fractions, wherefore I wholly refer you to the Rules of Fractions before most plainly taught, and so I end with Proportions.

Notwithstanding I had almost forgotten to set downe one necessary Rule more touching Proportions, which was also omitted in the first Impression, that is, how to find out the meane Proportionall number betwixt any two numbers given, which Rule is thus: Multiply the two numbers given the one by the other, and the square Root of the Product shall be the meane Proportionall number betwixt them. As for Example, suppose the two numbers given to be 9, and 36, which being multiplyed the one into the other, the Product will be 324, the square Root whereof is 18, and that is the meane Proportionall number betwixt the foresaid numbers given. Now how to take the square Root of any number, is plainly taught in the 24 Chapter ner following.

Chap. XXIIII.

How to find out the square Root of any number.

WHat is a square Root?

It is any Digit or any other number, which being multiplyed into it selfe bringeth forth a square number, as 4 being multiplyed in it selfe, in saying 4 times 4 maketh 16, which is a square number, the Root whereof is 4. And to find out the square Root of any number be it square or not square, you must do thus: First having set downe the number proponded which must consist of three figures at the least, set a prick under the first Digit or Cypher of the said number on the right hand, that done, prick every other figure thereof to-

wards the left hand, leaving alwayes one voyd ſpace betwixt every two pricks, as you ſee here done in this number 475643, and looke how many pricks there bee, ſo many figures ſhall you have in the Quotient, and this differeth nothing in a manner from Diviſion, the order in working is thus, firſt ſeeke out a Digit, which being multiplyed in it ſelfe, may take away that which ſtandeth over the laſt prick on the left hand, or as much thereof as may bee, and ſet the Digit in the Quotient, that done, double the ſaid Quotient, then conſider whether the double do conſiſt of one figure or of many, if of one only, then place that in the next voyd ſpace on the right hand, but if the double do conſiſt of many figures, then take the firſt figure thereof, that is to ſay, that figure or Cypher which ſtandeth in the firſt place of the double, and place that in the foreſaid next voyd ſpace towards the right hand, and next to that place all the reſt of the figures of the ſaid double orderly towards the left hand, that done, ſeeke another Digit, which being multiplyed in it ſelfe together with the double number, may take away all that which ſtandeth right over it, or as much thereof as may be, which Digit you muſt not only put in the Quotient to his fellow, but alſo ſet downe the ſame right under the next prick on the right hand, and then multiply that Digit together with the double whereto it is joyned, by the ſaid laſt found Digit ſet downe in the Quotient, the Product whereof you muſt ſubtract out of the upper number, ſtanding right over the foreſaid prick, and if there be any remainder write it above, canceling the other, and looke how many pricks there be, ſo many times muſt you double the Quotient, obſerving alwayes like order of working as before, which you ſhall more plainly underſtand by theſe examples following, and for your better underſtanding thereof, I will firſt give you an example of three figures only, as thus, 464, here to find out the ſquare Root of this number, you muſt firſt prick the ſaid number in ſuch order as is before taught, and then ſeeke out ſome ons of the 9 Digits, which being multiplyed in it ſelfe, may take away the firſt 4 on the left hand, or as much thereof as may be, which you ſhall find to be 2, which 2 being ſet in the

Quotent,

Quotient, multiply the same in it selfe in saying 2 times 2 maketh 4, which 4 doth cleane take away the first 4 standing over the last prick on the left hand, and therefore cancell the 4, that done, double the Quotient which maketh 4, and set the same in the next voyd space on the right hand under the figure 6 thus:

And then seek out some Digit, which being multiplyed in it selfe together with the doubled, may take away the Summe of 64 that remayneth, which by asking how many times 4 is comprehended in 6, you shall find to be but 1, which you must not only put in the Quotient, but also set downe under the first prick on the right hand thus.

$$\begin{array}{l} \cancel{4}64\ (2 \\ \cdot 4 \end{array}$$

So as you shall make the lower Summe to be 41 which is to be multiplyed by the last found Quotient which is 1, wherefore you must say that 1 time 41 is 41, which being taken out of 64. there remayneth 23, so as the Root is 21, and the remaynder 23, as the example sheweth.

$$\begin{array}{l} \cancel{4}64\ (21 \\ \cdot 41 \end{array}$$

And to know whether you have done well or not, multiply the Quotient in it selfe, and if there be any remaynder left, adde that unto the Product of the multiplyed Quotient, and if you find the Summe thereof to be like to the first, then you have done well, if it be not like, then you have erred, the Product of this Quotient being multiplyed in it selfe amounteth unto 441, whereto by adding the remaynder which is 23, you shall find the Summe to be like unto the first number that is 464, whereof you sought the square Root. But if such number do consist of many figures, then in working you must double the Quotient once, twice, or thrice, according as the number doth require, as you shall more plainly perceive by this example following, as let 50|467 bee the number whereof you would know the square Root, here having pricked this number in such order as before, first seeke out some Digit, which being multiplyed in it selfe may take away the Digit or number standing right over the last prick on the left hand, which you shall find to be 2, and having set downe the said two in the Quotient, say 2 times 2 maketh 4, which being

$$\begin{array}{l} 23 \\ \cancel{4}\cancel{6}\cancel{4}\ (21 \\ \cdot \cancel{4}\cancel{1} \end{array}$$

subtracted from 5, there remayneth 1 which you must set over 5, and cancell the said 5, as this example sheweth.

That done, double the Quotient, which maketh 4, and set downe the 4, in the next space towards the right-hand, under the Cypher, as you sée in the former example, then séeke out a Digit, which being multiplyed in it selfe together with the double Quotient, may take away all that which standeth over the second prick towards the left-hand, or as much thereof as may be, which Digit by asking how many times 4 is comprehended in 10, you shall find to be 2, then set downe the said 2, not only in the Quotient, but also under the second prick towards the right-hand, which together with the former 4, maketh 42, as you sée in this example.

```
 1
5 0467 (2
 4
```

Then multiply that 42, in a place by it selfe, by the figure 2 last set downe in the Quotient, and it will make 84, which 84 being taken out of 104, there remayneth 20, which you must set over the second prick and cancell the 104, and also the 42 beneath, as you sée in the former example: that done, double the whole Quotient, which maketh 44, whereof set the first 4 in the next voyd space towards the right-hand under the figure 6, and the other under the second prick towards the left-hand, then aske how many times 4 is comprehended in 20, and you shall find 4, which you must not only set downe in the Quotient, but also under the next and last prick on the right-hand, & so the nether sum shall be 444, which being multiplyed by the Digit 4 last set downe in the Quotient, will make in all 1776, which being taken out of the upper number, which is 2067, there will remaine 291, so as the square Root of the first number is 224, and the remainder 291, as you may sée in this example.

```
 1 20
5 0 4 67 (22
   4 2
-----
 84.
```

The

The ſquare Root whereof being multiplyed in it ſelfe amounteth unto 50176, whereto if you adde the remainder which is 291, the ſumme whereof will bee like unto the firſt number which is 50467.

Moreover you have to note, that if you find any number out of the which your Quotient being doubled cannot bee ſubtracted, then you muſt ſet downe a Cypher in the Quotient, and proceed to the next prick on your right hand, as in this example following.

In which example the firſt found Digit ſet in the Quotient is 6, which being multiplyed in it ſelfe maketh 36, and cleane taketh away the firſt number, ſtanding over the laſt prick on the left hand, that done, I double the Quotient, which maketh 12, and I place the 2 in the next voyd ſpace towards the right hand, and the 1 next to that towards the left hand orderly, as in this example.

Over which 12, there is but 6 remayning, ſo as I cannot take 12 out of 6, and therefore I cancell the 12, and ſet downe a Cypher in the Quotient, ſo as the Quotient is now 60 which being doubled maketh 120, the Cypher whereof I ſet downe in the next voyd ſpace on the right hand, and the other two Digits orderly towards the left hand, as you ſee in this example.

Then I aske how manytimes 1 is contained in 6, and I find 5, which I ſet downe in the Quotient, and alſo under the laſt prick on the right hand, ſo as the nether number is 1205, which I multiply apart by it ſelfe by the 5 laſt ſet downe in the Quotient, the Product thereof is 6025, which being taken out of the upper number which is alſo 6025, there remayneth nothing, as this example ſheweth.

Wherefore I find the foreſaid number to be a juſt ſquare number, the Root whereof is 605, which being multiplyed in it ſelfe maketh 366025, a number like unto the firſt number.

Chap. XXV.

The use of the Square Root in setting of battels.

THe knowledge of finding out the square Root of any number, is very necessary for a Sargent Major in the field, that he may the more readily set and range his Squadranes of battell: and therefore I thinke it not amisse, to give them here certaine examples of such manner of squares as the Italians were wont to use in my time, which is foure manner of wayes.

Whereof the first teacheth how to set the battell square of ground, the second a square battell of men, the third a long square battell which we call the Hearse battell, and the fourth teacheth how to set a battell of so much and a third.

Shew by what Rule these battels are to be set?

Before I set downe any Rules, you have to understand that all numbers given or supposed, are not meet for such purpose, but only such as may be divided into three equall parts, without leaving any remainder: for otherwise all the Rules following take no place, and therefore when any number is given you to be brought into any one of the foresaid foure formes of battell you shall do well first to divide the number by three, and if there be any remainder, to reject it, and to take the rest for your number. As for example, A Sargent Major is commanded by his Generall to set a battell square of ground, appointing him thereunto 1345 men, here the Sargent by dividing that number by three, findeth in the remainder one man too many, which being only rejected, all the rest of the number which is 1344 is fit for his purpose.

How

How to ſet a battell ſquare of ground, called in Italian Battaglia quadra diterreno.

NOw ſhew the order of ſeeting ſuch kinds of battells? The order is thus, firſt double the number given, then take the ſquare Root thereof, and that Root ſhall be the Front: that done, divide the number given by the ſaid Root, and the number in the Quotient ſhall be the ſlancke, now if you multiply the Front and the ſlancke the one into the other, you ſhall have your whole number firſt given, unleſſe there be ſome remainder left in the former Diviſion, which remainder muſt be added to the Product to make up the Summe. But if you would know how many men are to bee put in a ranke, and alſo how many ſuch rankes you muſt have, then divide the Front into 3 parts, and the Quotient will ſhew you how many men you are to have in a ranke, that done, multiply the ſlancke by 3, and the Product thereof ſhall be the number of the rankes: As for example, take the number before given, which was 1345, which being divided by three, you find the remainder to bee 1, which you muſt reject, and take all the reſt which is 1344, & that being doubled maketh 2688, the ſquare Root whereof is 51, by which Root you muſt divide the number given, Videlicet 1344, and the Quotient will be 26, which ſhall be the ſlancke, now if you divide the foreſaid Front 51 by 3, the Quotient will ſhew how many ſhall be in a ranke, that is 17. Finally, if you multiply the ſlancke, which is 26 by 3, the Product thereof will be 78, which is the juſt number of the rankes which you muſt have.

How to ſet a battell ſquare of men called in Italian Battaglia quadra d'huomini.

TAke the ſquare Root of the number given, and that ſhall be both the Front and the ſlanck, which if you multiply together the Product therof will make up the number firſt given, and if you would know how many men are to be put in a ranke,

and how many such rankes you must have, then do as you did before in dividing the Front by 3, and multiplying the flancke by 3.

How to set a long square battell which we call the Hearse battell and is called in Italian, Battaglia dun tanto emezzo.

TO the number given, adde the just halfe thereof, and the square Root of that sum shal be the Front, by which Root or Front if you divide the first number given, the Quotient shall be the flancke: as for example, let the number given be 6144, the halfe thereof is 3072, which being added to the given number 6144, maketh in all 9216, the square Root whereof is 96, which shall be the Front, by which Root or Front if you divide the given number 6144, you shall find in your Quotient 64, and that shall be the flancke, that done divide the foresaid Front 96 by 3, and you shall find in the Quotient 32, which number sheweth how many men shall be in a rancke, then multiply the foresaid flancke 64 by 3, and the Product thereof shall be 192, which is the number of the rankes.

How to set a battell of so much and a third, called in Italian, Battaglia dun tanto & dun terzo.

TO the number given, adde the third part thereof, and the square Root of the summe shall be the Front, then divide the first number given by the foresaid Root or Front, and the number in the Quotient shall be the flancke: as for example, let the number given bee 1575, the third part whereof is 525, which being added to 1575, maketh in all 2100, the Root whereof is 45, which must be the Front, by which Root or Front if you divide the number given which was 1575, the Quotient wil be 35, which shall be your flancke, then by dividing 45, which is the Front by 3, the Quotient will shew you how many men you must have in a rancke, that is, 15. Againe by multiplying the flancke which is 35 by 3, the Product thereof will be 105, which is the number of your rankes.

CHAP.

Chap. XXVI.

How to find out the Cubique Root of any number.

AS the ſquare Root is ſaid to be a number, which being multiplied in it ſelfe, doth make a ſquare ſuperficiall number, having only length, and breadth, ſo the Cubique number, is a number which being firſt multiplyed in it ſelfe, and the Product therof being againe multiplyed by the firſt number, doth make a Cubique or Corporall number, having both length, breadth, and depth: As for example, 2 times 2 maketh 4, and 2 times 4 maketh 8. Againe, 3 times 3 maketh 9, and 3 times 9 maketh 27, and ſo you may deale with all the reſt of the 9 Digets, and make thereby a Table containing both the ſquare numbers and Cubique numbers of every Root, conſiſting of any one of the 9 Digets, like unto this Table here following.

Roots	Squares	Cubiques
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729

When having to find out the Cubique Root of any number that is greater then 1000, (for leſſer it cannot be to worke upon) then ſet a prick under the firſt figure on your right hand, and ſo proceed towards the left hand, omitting alwayes 2 figures betwixt every two pricks, as in this example, 41063625, and looke how many pricks there be, ſo many figures ſhall you have in the Quotient: That done, you have to find out three ſeverall numbers, firſt ſuch a Cubique number as will cleane take away the number which ſtandeth right over the head of the laſt pricke

on

on the left hand, or so much thereof as may be, which you may easily find in the foresaid Table: The Root of which Cubique number, you must set downe in the Quotient, the second number which you have to find is a number called the Triple, which is easily had by tripling the Quotient, and the third number is called the Divisor, which you shall most readily find by multiplying the Quotient into the Triple, both which numbers are to be placed in such order as followeth. First then, having found the Cubique number, as is before said, and taken the same out of the number standing over the last prick on the left hand, write the remainder, (if there be many) over his head, cancelling that which is under it, and then place the Root of the said Cubique number in the Quotient: that done, Triple the Root set downe in the Quotient, and that shall be the Triple which you must place in the second void space on the left side of the next pricke which is on your right hand, then multiply that Triple by the Quotient, and the Product thereof shalbe the Divisor, which must be placed right under the Triple one figure shorter towards the left hand: that done, draw a line as you see in the example hereafter following, and worke thus: First aske how many times the first figure of the said Divisor is contained in the number standing right over his head: and having found an apt Digit for the purpose, put that Digit in the Quotient to his fellow: that done, multiply the said Digit into the Divisor, and set the Product thereof right under the Divisor. Secondly, multiply the said Digit in himselfe, & againe, the Product thereof into the Triple, and set the sum that comes thereof right under the Triple: Thirdly, multiply the said Digit in it selfe Cubically, and set the Product thereof lowest of all, right under the next prick on the right hand: that done, draw a line, and by addition bring all the foresaid three Products into one summe, placing every figure so as you may easily take the said sum out of the upper number, whereof you seeke the Cubique Root, and write the remainder (if there be any) over the head, and if such summe will not be subtracted out of the upper number, then you must seeke out a lesser Digit, reserving still the former Triple and Divisor, and if there be more pricks in the

number

mmber than two, then you must for every prick find out a new Triple and a new Divisor by tripling the whole Quotient, and by working continually in like order as the Rule before teacheth, which Rule you shall most plainly perceive by example thus: first then having to find out the Cubique Root of the number before mentioned, which is, 41063625 marked with pricks as before is taught, resort to the Table wherein you shal find such a Cubique number as will take away as much of the 41 as may be, which is 27, the Root whereof is 3, for all the other Cubique numbers in the Table are either too small or too great, and therefore you must alwayes have due consideration thereof. Then take 27 which is the Cubique number, out of 41, and there remayneth 14, which remaynder you must set over the last prick on your left hand, cancelling the 41, as in this example.

Then triple the Quotient which maketh 9, which you must set downe under the figure 6, next unto the second prick which is on your right hand: Then to find out the Divisor, multiply the Triple 9 by 3, which is the Quotient, and the product thereof shall be the Divisor: which Divisor you must place right under the Triple one figure shorter towards the left hand, that done, draw a line as you see in this example.

14
4 1 0 6 3 6 2 5 (3

Now having orderly set downe the Triple and the Divisor, aske how many times 2, which is the first figure of the Divisor, is contained in 14, standing right over his head, and remember to make choyce of such a Digit, as may not cleane take away the whole number at the first, but rather leave so much as the said Quotient having afterward to bee multiplyed divers wayes, as shall be said hereafter, may take away the rest or as much thereof as may be, as in this example following, you shall find the aptest Digit for this purpose to be 4, which you must put in the Quotient, whereby the Quotient shall be now 34, that done, you must first multiply this last Quotient

Triple-- 9
Divisor---27

which,

which is 4, into the Diviſor 27, the Product whereof is 108, which you muſt ſet downe right under the Diviſor, beneath the line already drawne, as you ſée in this example.

Secondly, you muſt multiply the ſaid 4 in it ſelfe Quadrantly which maketh 16, then multiply that 16 by the Triple 9, the Product whereof is 144, which is to be ſet right under the Triple, as in this example:

Thirdly, you muſt multiply the ſaid 4 in it ſelfe Cubically, which maketh 64, which is to be ſet right under the next prick on the right hand beneath the former ſummes, as you ſée in this other example.

So as every one of the foreſaid three Products doe extend one further then another by one Figure towards the right hand, as you ſée in the former example: Now theſe Products being thus placed, draw another line and bring all the three ſeverall Products contayned betwixt the 2 lines into one ſumme by Addition, and you ſhall find the totall ſumme to bee 12304, which being ſubtracted out of the number ſtanding right over it which is 14063, there will remaine 1759, which you muſt ſet downe above as you ſée in this example.

Now to proceed with the next: you muſt find out a new Triple and a new Diviſor.

How is that done?

Thus, multiply the whole Quotient which is 34 by 3, the Product whereof is 102, and that is the Triple, which is to bee placed

14
4 X 063625 (34
Triple--||,
Diviſor--27
108
14
4 X 063626 (34
Triple-||
Diviſor--27|
108|
144
14
4 X 063625
Triple 91
Diviſor 27||
108||
144|
64
1
X 4759
4 X 063625 (34
Triple 9
Diviſor 27||
108||
144|
64
12304

placed in the next voyd space hard by the next prick that is on your right-hand. Then multiply the whole Quotient into the last found Triple 102, and the Product thereof which is 3468 shall bee your Divisor, which is to be placed under the Triple one figure shorter toward the left-hand, as you see in the example following: which two numbers being thus found and rightly placed, draw a line, then aske how many times the first figure of your Divisor which is 3, is contayned in the number right over his head which is 17, and you shall find it to be 5 times contayned therein, wherefore set downe 5 in the Quotient, that done, multiply 5 in the Divisor and place the Product thereof which is 17|340, right under the Divisor beneath the line, as you see in this example.

Secondly, square the same 5 that is to say, multiply it in it selfe, and that maketh 25, which 25 you must multiply againe into the Triple 102 the Product whereof is 2550, which number you must set right under the said Triple beneath the line. Thirdly, multiply the said 5 in it selfe Cubically, which maketh 125, and place that right under the first prick on the right hand, and draw a line: that done, adde all the foresaid 3 Products together, and you shall find the totall summe thereof to bee 1759625, which being subtracted out of the upper number, there remayneth nothing, as this example plainly sheweth.

```
   1
 1 4759
 4 1 0 6 3 625 (345
Triple   102
Divisor  3468
-------------
         17340
```

Whereby you may conclude that the foresaid number whereof you sought the Root, is a perfect Cubique number, for if you multiply the whole Quotient in it selfe Cubically it will produce the selfe same number whereof you sought the Root. But note, that if in making the first Subtraction, the first Divisor is not to be found in the upper number, then

```
   1
 1 4759
 4 1 0 6 3 625 (345
Triple   102
Divisor  3468
-------------
         17340
          2550
           125
-------------
        1759|625
```

you:

you must set a Cypher in the Quotient, that done, triple the whole Quotient, and place the said Triple under the figure which is next to the next prick on the right hand, working in such order as before, and to aviod confusion, having to deale with a multitude of numbers it shall not bee amisse at the finding out of every new Triple, and Divisor, to set the remainders in severall places by themselues, and to worke then in the selfe same order that is before taught: also note that if you have to deale with a few numbers, and that the Divisor cannot bee subtracted out of the number standing right over his head, then you must set a Cypher in the Quotient, and so you have done. As for example, having to take the Cubical Root of 8597, here I find 2 to be the Quotient, which being Cubically multiplyed, doth cleane take away the 8, and now according to my former Rule 12 must be my Divisor, which because I cannot take it out of 5, I set downe therefore a Cypher in the Quotient, so as the Quotient is now 20, which is the Cubicall Root of the foresaid number, for if you multiply 20 Cubically and adde thereunto 567 which is the remaynder, the summe thereof will bee like unto the first number, as you may see in this example.

		20
8567.		20
6	Triple (20	400
12	Divisor	20
		8000
		567
		8567

Chap. XXVII.

Of Astronomicall Fractions.

Because the use of these Fractions is very necessary for those that have to calculate the motions of the Stars and the Difference of time, I thought good to shew here how the same are to bee added, subtracted, multiplyed, and divided, for the measure of Time falleth not out

out alwayes to be a whole yéere, month, day, or houre, nor the moving of the Celeſtiall bodies are to be meaſured alwayes by whole Circles, Signes, or whole Degrées, and therefore to have an exact meaſure of ſuch things it was thought beſt by the Ancient Writers to divide all whole things called Integra into the leaſt parts that might be, for which purpoſe no Number was thought ſo mæt as 60, for there is no number under 100, that receiveth ſo many Diviſions as 60, which may be divided many ſundry wayes, that is, by 2, 3, 4, 5, 6, 10, 12, 15, 20, and by 30, and therefore they divide every whole thing that had no uſuall parts into 60 minutes, and every minute into 60 ſeconds, and every ſecond into 60 thirds, and ſo forth into 60 fourths, fifths, ſixths, ſevenths, eighths, ninths, and tenths, and further if néed were, but that ſeldome chanceth. And you have to note that minutes are marked with one ſtrâk over the head, ſeconds with two ſtrækes, thirds with thræ ſtrækes, and ſo forth thus, $23', 6'', 7''', 8''''$, &c. which do ſignifie 23 minutes, 6 ſeconds, 7 thirds, and 8 fourths.

Of Addition.

WHat is to bee obſerved in adding of theſe kinds of Fractions?

Firſt, that you bring Integrums to Integrums, and Fractions to Fractions, that be of like Denomination, beginning alwayes with the leaſt on the right hand, and if the ſumme of ſuch Addition do amount any where to the number of 60, or above 60, then you muſt looke how many 60 are comprehended therein, and for every 60 adde one to the next greater Fraction that is on the left hand, obſerving ſtill that order untill you come to the Integrums, of which Integrums it is alſo neceſſary to know their value, that is to ſay, what parts they contayn, and what Denomination thoſe parts have. As for Example, if you adde common Signes ſuch as the 12 Signes of the Zodiake be, then every Signe containeth 30 degrées, ſo as every ſumme exceeding 30, is to be divided by 30, but if they bee

Physicall signes, whereof 6 do make a whole circle, such as be set downe in the Table of Alphonsus, then the summe of those degrees is to be divided by 6. Moreover, so often as the summe of the common signes do exceed 12, or the summe of the Physicall signes do exceed 6, the overplus is alwayes to be rejected, and the remainder to be set in the place of the signes, as you may see in this Example following, wherein Seconds are reduced to Minutes, Minutes to degrees, and finally degrees to signes.

An Example of Addition consisting of Signes, Degrees, Minutes, and Seconds.

Signes	Degrees	Minutes	Seconds
9	19	1	19
0	0	35	16
11	29	33	5
9	29	28	11
0	11	49	40
		4	56
8	0	42	27

Summa totalis.

In which Example beginning first with the seconds, because they are here the least Fractions, you shall find by addition that the summe of them amounteth to 147, which being divided by 60, you shall find in the Quotient 2, and the remainder to be 27″, which remainder you must set downe under the Collum of Seconds, keeping the Quotient which is 2 in minde to be added to the Collum of Minutes, the Summe whereof is 162, which being divided by 60, you shall find in the Quotient 2 degrees, and the remaynder to be 42′ which is to be set downe under the Collum of Minutes, and the Quotient 2 kept in minde to be added to the Collum of degrees, the Summe whereof is 90 degrees, which being divided by 30, (because that 30 degrees do make one common whole signe) you shall find in the Quotient 3 signes and no remaynder: wherefore you must set downe a Cypher under the Collum of degrees, and adde three signes to the Collum of signes, the summe whereof is 32, which if you divide by tirice 12, which maketh 24, you shall find the remaynder to be 8 signes, which

is

is to be set downe under the Collum of signes, for the Quotient here is to bee rejected, according to the Rule before given, so as the totall Summe of this Addition is 8 Signes, 0 Degrees, 42', 23".

23	2	1	0

Another Example of Dayes, Houres, Minutes, and Seconds, to be added together.

	Dayes	Houres	Minutes	Seconds
	21	14	32	11
	16	16	19	41
	8	16	20	30
Summa totalis.	46	23	22	22

In this Example one selfe order as in the other before is to bee observed as touching the Seconds and Minutes, for the exceeding number in both, are divided by 60, but when you come to the houres, you must divide that Number by 24, because that is many houres do make a whole day, and having set downe the remaynder under the Collum of houres, adde that one day which was in the Quotient, unto the Collum of dayes, and so you shall find the totall Summe to be as the Example above sheweth.

Of Subtraction.

WHat order is to be observed in Subtraction?

The selfe same that was before observed in Addition, so as you alwayes remember, that when you have to take a great number of Fractions, as of Minutes, Seconds, Thirds, and such like, out of the lesser Number of Fractions, to borrow 60, and having set downe the remaynder, to adde the one borrowed, unto the nert Collum on the left hand, for there the 60 borrowed is but one, but if you have to deale with Degrees, which are counted Integrums, then you must borrow but 30, for so many Degrees do make one Signe, and if you have to subtract houres then you must borrow 24, for so many houres do make one day, as by the Example here following, you shall more plainly perceive.

Signes	Degrees	Minutes	Seconds
8	0	42	27
0	1	9	53
7	29.	22	34

In this Example because you cannot take 53″ out of 27″ you must borrow one minute from the next Collum on the left hand, which one minute is 60″, which being added to 27, do make in all 87″, out of which if you take 53′, there will remaine 34″, that done, you must pay home the minute which you borrowed by adding the same unto the next 9 on the left hand, which maketh 10, then say, take 10′ out of 42′, and there remayneth 32′, which being set downe, proceed to the next. But here to take one out of none that cannot be, and therefore borrow one whole signe from the Collum of signes, which is 30 degrees, from whence take one and there remayneth 29, which being set downe, take the one which you borrowed out of 8, and there remayneth 7, so as the whole remaynder of this Subtraction is 7| signes, 29| degrees, 32|, and 34″, as the former Example sheweth.

Multiplication.

THough there bee more difficulty in multiplying and dividing Astronomicall Fractions, than in adding or subtracting them, yet the greatest difficulty thereof, chiefly consisteth in the finding out the true Denomination of the Products, for first as touching Multiplication, you must multiply every number of the multiplyer into all the particular numbers of the summe that is to be multiplyed, and then severally to adde together the Products that be of one selfe Denomination, and whatsoever in that Addition ariseth to the number of 60, or exceedeth 60, it is to be reduced by the Seragenary Division into the greater summe, so shall you collect the whole summe of the Multiplication. But you have to note by the way, that if there be any Integrums of divers Denominations, in your Multiplyer: That then such Integrums must be reduced to one

one selfe kind of Integrums: As for Example, suppose that you would multiply the daily motion of the Moone, which according to Alphonsus Tables is 13 degrees, 10′, 35″, and 1‴, by 29 dayes, 12 houres, 44′ and 3″, here because there be in this multiplyer Integrums of Diuers Denominations, that is to say, dayes and houres, you must therefore reduce the same into one selfe Denomination, before that you can make your Multiplycation.

How is that to be done?

By multiplying every Number of the said Multiplyer by 5 and then by halfing the Product thereof, by which halfing you shall reduce Minutes to Seconds, and Seconds to Thirds, and so forth, to the smallest Fractions of all, and if any Product do amount to 60, or exceed 60, then you must diuide the Product by 60, the remaynder whereof is to be set under his proper Denomination, and you must keepe the Quotient in mind to adde the same to the next greater Number, as this Example or Table sheweth: in the Front whereof I haue set downe in seuerall spaces, not only the Denominations of the two Integrums, as dayes and houres, but also the Denominations of so many Fractions as I thinke meet to serue my turne, under which Front I place the foresaid Multiplyer, and then draw a Line as you see in this Example following.

Denomination.	Dayes	Houres	′	″	‴	⁗
The Multiplyer to bee reduced into one selfe Denomination.	29	12	44	3		
The Products of the reduction.		29	21	50	7	30

Now beginning first on the right hand with the least Fraction of the said Multiplyer, which is 3″, I multiply 3 by 5, which maketh 15, the one halfe whereof is 7″, and halfe a third, which is 30‴, wherefore I set downe the sayd 7″, and 30‴ vnder their proper Denominations as you see in the Example above, then I multiply 44′ by 5, the Product whereof is 220, and the halfe thereof is 110, which being diuided by 60, the Quotient is 1′, and the remaynder 50″, which remaynder I set under his proper Denomination, keeping the Quotient still in mind, that

done, I multiply 12 houres by 5, the Product whereof is 60, and the halfe of that is 30′, whereunto by adding the one which I had in mind, I make it 31′, and so I set it down under his proper Denomination, and because there be no more Fractions to be multiplyed, I set downe on the left hand the Integrum 29, and by this meanes I have brought the foresaid Multiplyer to one selfe Denomination and to one kind of Integrum, that is to say, to 29 dayes, 3′, 50″, 7‴, 30⁗, which now being the greater number is to be set above, and to be multiplyed by the foresaid daily motion of the Moone, that is, 13 degrees, 10′, 5″, and 1‴, but to the intent that in multiplying these Numbers together, you may set every Product in his true place, that is to say, under his proper Denomination, it shall not be amisse in the Front of your worke to set downe two rowes of Numbers, whereof the first must containe so many Denominations or Fractions as you thinke good, as Minutes, Seconds, Thirds, Fourths, and so forth, marked with strokes and vulgar Numbers, and the second row shall be the naturall order of Numbers, written in Arithmeticall figures as this Table sheweth.

Integra.	′ ″ ‴ ⁗	v	vi	vii	viii	The Denominations.
0	1, 2, 3, 4, 5, 6, 7, 8,					The naturall Numbers.

Under which Table you must first set the Number that is to be multiplyed, and right under that the Multiplyer in such sort as every particular Product may be placed under his proper Denomination, and then draw a Line as you see in this Example following, and when you have multiplyed two Numbers the one into the other, and know not where to place the Product, then marke under which of the naturall Numbers in the Front, the said two Numbers, that is to say, the Multiplyer and the Multiplycand do stand: That done, adde those two naturall Numbers of the Front together, and the Summe thereof will shew you under what Denomination the Product is to be placed, as in this Example.

Integ.	′	″	‴	⁗	v	vi	vii	Denominations.
0	1	2	3	4	5	6	7	Naturall Numbers.
29	31	50	7	20				The Multiplicand.
13	10	35	1					The Multiplyer.
			29	31	50	7	30	
		1015	1085	1750	245	1050		The severall Products.
	290	310	500	70	300			
377	403	650	9	390				
389	6	24	2	31	1	37	30	The generall Product or totall Summe.

In which Example I first multiply 1‴ into 20⁗, the Product whereof is 30ᵛⁱⁱ, which must be set under the Denomination vii, because the two naturall Numbers, that is, 3 standing over 1 the Multiplyer, and 4 standing over 30 the Multiplicand, being added together do make 7, appointing to the Product his proper Denomination, then multiply againe the same 1‴, into 7‴, the Product whereof is 7ᵛⁱ, which must be set under the Denomination vi, because 3 standeth over both their heads, and therefore must be taken 2, that is to say, for each Number 3, which being added together do make 6, appointing thereby to the Product his proper place of Denomination, that done, multiply the said 1‴, into 50″, the Product whereof is 50″, which must bæ set under the Denomination v, for the 3 which standeth over 1, and 2 over 50″, being added together maketh 5, appointing to the Product his proper place of Denomination, then multiply the said 1″ into 31′, the Product whereof is 31‴, here the two naturall Numbers, that is to say, 3 standing over 1″, and 1 over 31′, being added together, do make 4, appointing to the Product his proper place of Denomination, then multiply the foresaid 1″ into 29 Integrums, the Product whereof is 29‴, and must be set under the Denomination ‴, because 3 standeth over 1‴, but 29 being an Integrum, hath no naturall Number standing over him but a Cypher, thus having gone throughout al the Numbers of the Multiplycand, with the first Number of the Multiplyer, proceæd in like order with the second Number of the Multiplyer, which is 35″, which being multiplyed into 30‴ maketh 10500ᵛ,

to be set under the Denomination vi, because 2 standeth over 35″ the Multiplyer, and 4 over 50‴ the Multiplicand, which 4 and 2 being added together maketh 6, then multiply 35 by 7‴ which maketh 245ᵛ, which you must set down under the Denomination v, because 2 and 3 maketh 5, that done, multiply 35″ by 50″, which maketh 1750⁗, which you must set downe under the Denomination ⁗ because the Multiplicand and the Multiplyer are both under the Denomination ″, which being twice repeated maketh 4, then multiply 35″ by 31′ and that maketh 1085‴, which you must set downe under the Denomination ‴, because 1 and 2 maketh 3, finally multiply 29 Integrums by 35″, and that maketh 1015″: thus as you have gone through with two Numbers of your Multiplyer, so proceed in like order with the other two Numbers of the Multiplyer, which is 10 and 13, and when you have ended your Multiplication, and set every Product in his proper place, and so as every figure may stand ons right under another, to avoyd confusion when you come to Addition, (to which end the spaces of Collums had need to be the larger) then draw a Line under all the Products, and beginning on the right hand, adde all the Products contayned in every severall Collum together, and if the Summe of any such particular Addition do arise to the Summe of 60, or exceed the Number of 60, then divide that Summe by 60, and set downe the remaynder, keeping the Quotient in mind, to be added to the Product of the next Collum on the left hand, so shall you find the totall Summe of your multiplication to be 389 degrees, 6′, 24″, 2‴, 3⁗, 12ᵛ, 37ᵛⁱ, and 30ᵛⁱⁱ, as the former Example plainly sheweth. Now if you divide 389 degrees by 30, because every common signe contayneth 30 degrees, you shal find your totall Sum to be 12 signes, 29 degrees, 6′, 24″, 2‴, 31⁗, 12ᵛ, 37ᵛⁱ, and 30ᵛⁱⁱ, and so much the Moone runneth in the space of 29 dayes, 12 houres, 44′, and 3″ of an houre, which is her full resolution betwixt every two changes, but for as much as it chanceth as well in this Example as in many others like, that Integrums of two sundry Denominations are propounded in the question, it may very well be doubted with what Denomination the Product of such Multiplication is to be named, as in this

Example

Example having multiplyed Time by Motion, a man may alke whether the Product shall be named dayes, or degrees, the resolving of which doubt dependeth upon the nature of the question propounded, for in the foresaid Example, because Time of daies do comprehend any certaine appointed Motion, therefore the Product of the Multiplication is to be referred to the degrees of Motion which are comprehended under Time, and not to Time, which comprehendeth Motion, wherefore this Product of Integrums viz. 389 signifieth here degrees and not dayes, so likewise when degrees and minutes are multiplyed by myles and minutes, the Product of such Multiplication taketh his name from myles, and not from degrees, because degrees do comprehend myles, for we say in matters of Geography, that every degree of the great Circle comprehendeth 60 myles: thus having spoken sufficiently of the Multiplication of Astronomicall Fractions, we will now proceed to the Division of such Fractions.

Of the Division of Fractions Astronomicall.

WHat is to be observed herein?

First, you must consider whether your Divisor be compound, or simple, I call that compound which contayneth Fractions of divers Denominations, and that simple which consisteth of Integrums, or in one whole Number of one selfe Denomination, wherein there is no difficulty, for then you have no more to do but to divide every particular Number contayned in the Dividend by that same Divisor, and to place the Product of every one under such Denomination, as the little Table of Denominations sheweth, and therefore it shall not be amisse to set the foresaid little Table over your Dividend even as you did in Multiplication: Also the Septagenary Progression is alwayes to be used, as well in Division as in Multiplication. Moreover, if your Divisor be not exactly contayned in the Dividend, then having multiplyed the Dividend by 60, you must adde to the Product thereof the next fraction following: As for Example, knowing by Alphonsus Tables, that the daily Motion of the Moone is 13 Degrees, 10′, 35″, 1‴, 15⁗, you would

would know how much she goeth in the space of an houre, here because that one day containeth 24 houres, the number must be 24 your Divisor, which is simple and not compound: first then set downe in the Front of your worke the row of Denominations only, and not the naturall Numbers, because they are not to be used in this way of Division, that done, right under the row of Denominations, place your Dividend, and right under that your Divisor, as you see in this example.

Degrees	′	″	‴	⁗	v	The Denominat.
13	10	35	1	15		The Dividend.
24						The Divisor.
	32	56	27	33	7	The severall sum. of every quotient.

In which example, because the Divisor 24 is not contayned in 13, therefore I multiply 13 by 60, which maketh 780, whereunto by adding the next Fraction on the right hand, which is 10′, the whole sum is 790′, which being divided by 24, the Quotient is 32′, which because they are minutes, I place them under the Denomination of minutes, and the remainder is 22′, which being multiplyed by 60, maketh 1320″, whereunto I adde the next figure, which is 35, and so the whole summe is 1355″, which being divided by 24, the Quotient is 56″, which I place under the Denomination of seconds, and the remainder of this Division is 11″, which being multiplyed by 60, maketh 660‴, whereto I adde the next Fraction which is 1‴, so that now the whole summe is 661‴, which being divided by 24, the Quotient is 27‴, which I set downe under the Denomination of thirds, and the remainder is 13‴, which being multiplyed by 60, maketh 780⁗, whereunto I adde the next Fraction, which is 15⁗, which maketh in all 785⁗, which being divided by 24, the Quotient is 33⁗, which I place under the Denomination of fourths, and the remainder is, 3⁗, which being multiplyed by 60, maketh 180v, whereunto having no Fraction to adde, I divide the same by 24, and so I find in the Quotient 7v, which I set under the Denomination of fifths, so as I find the hourely mo. of the Moone to be 32′, 56″, 27‴, 33⁗, 7v, and somewhat more, for I leave to deale any further with the smaller Fractions, that would still grow by multiplying the remaynders by 60,

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thinking this sufficient to shew you in what order you have to worke, to divide your Dividend by a simple Divisor, into as many small parts as you will: but if your Divisor bee compound, then the Division is to be done either by Reduction into the smallest Fractions, or without Reduction: which last way is very hard and tedious, and therefore I will only shew you how to make your Division, whereof the Divisor is compound by Reduction, and that by this one Example here following. Suppose then, that the Moone according to her owne course, which is from West to East is distant from some fixed Starre 36 degrees, 30', 24'', 10''', and 15'''', and that you would know in what time she will run that distance, according to her daily moving, which as hath bin said before, is 13 degrees 10', 35'', 1''', and 15'''', here to make this Division by Reduction, you must do thus: First reduce all the numbers of your Dividend into the smallest Fractions thereof, by the Seragenary Multiplication, and Addition of the next Fraction unto the Product of that Multiplication, that done, reduce all the numbers of your Divisor by like Multiplication and Addition into the smallest Fractions, so as the Dividend and the Divisor may bee both of one selfe Denomination, and divide the one by the other, even as they were Integrums, as in this Example you must first multiply 36 degrees by 60, and it will make 2160', whereto by adding 30', you make the whole summe of minutes to be 2190', which being multiplyed againe by 60, do make 131433'' whereto if you adde the 24'', the summe of seconds will be 131424'', and so proceeding still with the Seragenary Multiplication and Addition of the next Fraction, as you did before, you shall find the Dividend to be 473129451''''. Then in like order reduce your Divisor into the smallest Fraction, and you shall find the totall summe thereof to be 173766075''''. This Reduction being made, divide the Dividend by the Divisor, so shall you find in the Quotient two Integrums, that is to say, two dayes, and the remainder to be 132597365'''', which remainder if you multiply by 60, and divide the Product by the selfe-same Divisor, you shall have in the Quotient minutes, then multiply againe that remainder by 60, and divide the Product thereof by the same

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Divisor, and you shall have in the Quotient seconds, and so by observing still that order, you shall bring it into as small Fractions as you will: thus shall you find that the Moone, according to her dayly Motion, will run the foresaid space of distance that was betwixt her and the fixed Starre, in two dayes, 14 and 24.

CHAP. XXVIII.

How to divide Astronomicall Fractions, when the Divisor is greater than the Dividend.

THough by the last Chapter you may learne how to divide any number in Astronomical Fractions, whereof the Divisor is greater than the Dividend; yet I mind once againe to set down a generall Rule, to serve for such purpose, because it commeth often in use, in having to deale with Astronomicall Tables, and to give you example thereof: First then having to divide any number, whose Divisor is greater than the Dividend, do thus: multiply the greatest Denomination of the Dividend by 60, and if there be any Fractions annexed thereunto, which are of the next inferior kind, as minutes are to degrees, or to houres, and seconds to minutes, and thirds to seconds, &c. then adde them to the former Product: but if such Fractions be not of the next inferior kind, then let them stand as they are, untill you come to deale with them, and having divided according to the common Rule of Division, the first sum of the Dividend by the Divisor, multiply the Quotient into the whole Divisor, and subtract that Product out of the upper number if it may be, if not, then take the Quotient lesser and lesser, untill you can find such a number as will be subtracted out of the said upper number: and if there be any remainder left, then multiply that remainder by 60, not leaving to follow the former order of working, untill you have found the nearest exact Quotient that may be. And you have to

A generall Rule.

note,

note, that the Denomination of the first Quotient must be of the next inferior kind, to that Denomination which the Divisor hath: and to mak this Rule the plainer, I will set downe an example used by Scadius in the 115 Page of his Ephemerides, who to know the very instant of the full Moone the second of March, 1569, biddeth to divide the distance of the opposition, which was 8 degrees 46′ by the diurnall excesse of the Moones Motion from the Sun, which was then 13 degrees, 48′, which Divisor, because it is greater than the Dividend, you must according to the Rule before given, work thus: First multiply the greatest Denomination of the Dividend, which is 8 degrees, by 60′, the Product whereof will be 480′, whereunto by adding the Fraction annexed, videlicet, 46′, it maketh in all 526′, which is to be divided by 13 degrees, 48′, here in dividing the last Product first by 13 degrees, I find in the Quotient 39, which is one too many, considering that I must take out of the foresaid Dividend, 48′, as often as I did take out thereof 13 degrees, wherefore I set downe but 38 in the Quotient, and then the remainder will be 96, which because I may easily divide by the common Divisor 13 degrees, and 48′, I divide therefore that 96, first by 13, whereof the Quotient is 7, and the remaynder is 5, which I reduce into seconds, by multiplying that 5 by 6, the Product whereof is 300″, that done, I multiply 48′ by 7, the Product whereof is 336, which though it be somewhat too great a number to be taken out of 300, yet I let it stand, because it approcheth to a very nigh exactnesse, and by this meanes I find the whole Quotient to bee 38′, and 7″, and you have to note, that if after the first Quotient be set downe, there happen any remainder which is lesser than the Divisor, then you must set downe a Cypher in the Quotient, and remove your Divisor one place further, even as you do in Common Division, and then to worke as before.

CHAP.

Chap. XXIX.

How to find out the square Root of any Astronomicall Fractions.

THe greatest difficulty hereof consisteth in finding out the true Denomination of the Root, for if the Fraction be seconds, then the Root thereof are minutes, and if the Fraction be fourths, then the Root are seconds, for the Fraction must alwayes have such Denomination as may be halfed, seconds, fourths, and such like, the one halfe whereof giveth alwayes name to the Root, for if the Quotient be of thirds, you must first reduce them to fourths, before you can take the Root, and you must do the like with any other Fraction, whose Denomination is odde and not even. As for example, if you would take the Root of 43‴, here by multiplying these 43‴ by 60, you shall reduce them into 2580⁗, the Root whereof is 50′. Moreover, the Fractions wherewith you have to deale, are either simple or compound: if they be simple, and lesse then minutes, and therewith have even Denomination, and not odde, then you need to make no further reduction, but to worke as if you had to deale with whole numbers. As in seeking the square Root of 1600″, you find it to be just 40′, but if the number be compounded, that is to say, consisting of Integrums and Fractions, or of many Fractions having divers Denominations, then you must first reduce them all to the smallest Fraction that hath an even Denomination, before that you can take the Root. As for example, you would know the Root of 4 degrees, 25′, here you must by the Seragenary Multiplication, and Addition of the next Fraction reduce the degrees to minutes, and the minutes to seconds, as you were taught before in Division, and then to worke as you were wont to do in taking the square Root of whole numbers, and in so doing, you shall find the summe of seconds to be 15800″, the square Root whereof is 126′, which if you divide by 60, it will make 2 degrees, 6′. Another example, as to take the Root of 13

degrees

degrées, 42', and 45'', here by reduction as before, you shall bring the degrées and minutes to 49365'', the square Root whereof is 221', which being divided by 60, maketh 2 degrées, 42'. And thus I end with Astronomicall Fractions: which kind of Fractions though they be very learnedly and orderly taught by Reinoldus in the beginning of his Prutenicall Tables, yet in mine opinion, not in so plaine order, and so fit for every mans understanding, as I have here set them downe, according to the doctrine of Gemma Frisius, which being once learned, you shall the sooner attaine to the other. And without the knowledge of these Fractions, you can never truly calculate any thing out of the Astronomicall Tables, and therefore such Fractions are most necessary to be learned.

Notwithstanding, the better to calculate by the said Prutenicall Tables, or by any other Astronomical Tables, I must set downe here one thing more, which I had cleane forgotten in my first Edition, that is, to declare unto you, that in those Tables are often used two kinds of Denominations, whereof the one is called Sexagenæ, and the other Sexagesimæ, and that the Denomination Sexagenæ being set over any number, doth signifie, that the unite of the Integrum is multiplyed by 60. But the Denomination Sexagesimæ doth signifie, that the unite of the Integrum is divided by 60, and you have to note, that Sexagenæ have the like parts or Denominations that Sexagesimæ have, as Primes, ''. ''. '' and so forth, differing only in placing the said Denominations, for the Denominations of the Sexagenæ are alwayes set on the left hand of their Integrums, but the Denominations of the Sexagesimæ are set on the right hand of their Integrums. And to ease you of your paine as well in multiplying and in dividing, as also in taking the square Root of Astronomicall Fractions, I have thought good to set downe this Table here following, called the Sexagenarie Table, together with the description and use thereof, which Table is also set downe in my first Edition.

The

The description and use of the *Sexagenarie Table.*

THis Table consisteth of two figures, whereof the nether figure having foure Angles, is called in Latine Trapezium, marked with the Letters A. D. E. B. and the upper figure is a Triangle, marked with the Letters A. B. C. and each figure containeth particular Columnes of Numbers, serving to find out the Product of Astronomicall Fractions, being multiplyed one by another, and also the Quotients of the like Fractions being divided one by another, and also the square Roots of the said Fractions, for which purpose, the first Columne on the left hand containeth 59 Fractions, counting from 1 standing above, and so proceeding downward to 59, and are contayned betwixt A and D, and the foot of the said Trapezium containeth 30, counting from the left hand towards the right, which are contained betwixt D and E, and the rest of the numbers to make up 59. are to be found in the uppermost Front of the Triangle, proceeding from C towards A: you have to note also, that in the uttermost Columne of the Trapezium on the right hand, the numbers do proceed downeward from 30 to 59, that is, from B to E, and the numbers in the outermost Columne of the Triangle on the right hand, do proceed upward from 31 to 59, contayned betwixt B and C, both which do serve to fill up the first multiplyers, and multiplycands, for when you cannot find them in the Trapezium, then the outermost Columne of the Triangle on the right hand serveth to supply that want, and when you cannot find the said numbers in the Triangle, then the outermost Columne of the Trapezium

The Sexagenarie Table whereby the Products in Multiplication, the Quotients in Division, and the square Rootes are found in Astronomicall Fractions.

The Trapezium.

Place this between folio 80 and 81

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outermost Columne of the Triangle on the right hand serveth to supply that want, and when you cannot find the said numbers in the Triangle, then the outermost Columne of the Trapezium

pezium on the right hand, serveth to supply that want, and in seeking any Multiplyer or Multiplicand, either in the Trapezium, or in the Triangle, consider alwayes which way they are most readily found out, so as they may directly answere one against another.

All the rest of the numbers contained betwixt the two outermost Collums and are downe in square Angles, called common Angles do signifie either Products, or Dividends, or square numbers, according as occasion shall require. And the outermost Collums do signifie sometime multiplyers, sometime multiplicands, sometime Quotients, and sometime Roots. All which things you shall better understand by the Examples hereafter following.

The use of the Table.

By helpe of this Table you may more readily multiply and divide Astronomicall fractions, and also find out the square Root of such fractions, than by those Rules which I have heretofore set downe according to the doctrine of Gemma Frisius. And first, I will set downe an Example of Multiplication, then another of Division, and thirdly, one Example of finding out the square Root of the said Fractions, & let the Example of Multiplication be thus: Suppose that you would multiply 29 Degrees, 31′, 53″, 7‴, 3⁗, by degrees, 10′, 53′, 1‴, which you must set downe in such order as followeth, (that is to say) first the Denominations, then next under them the Multiplicand, and next under that the Multiplyer, and under them the severall Products, and lowest of all the totall summe of the said Products.

Here beginning with the first number of the multiplier on the right hand, which is 1‴, say thus, one times 30 is 30, which is to be placed under the Denomination of

In.	De	′	″	‴	⁗	v	vi	vii	Denominations.
	29	31	50	7	30				Multiplicand.
	13	10	35	1					Multiplier.
				24	31	50	7	30	The 1 Product.
		17	11	34	14	22	30		The 2 Product.
		4	55	18	21	15	0		The 3 Product.
		23	53	51	37	30			The 4 Product.
6	29	9	24	2	21	12	37	30	The totall Summe.

seven,

seuen, because the Denomination ouer 1 is 3, and the Denomination ouer 30 is 4, which being added together according to the Rule before set downe in the Chapter of Multiplication of Astronomicall Fractions do make 7, then say one times 7 is 7, which you must place under first Denomination. Againe, one times 50 is 50, which is to be set under the fift Denomination, because 3 and 2 maketh 5, then say one times 31 is 31, which is to be set under the fourth Denomination, for 3 and 1 maketh 4, then one times 29 is 29, which is set under the third Denomination, and thus you haue the first Product, then proceed with the nert Number of the Multiplyer towards your left hand, which is 35. and is to be multiplyed into 30, which to do readily, you must enter the Table with these two Numbers, and seeking in the first Collum of the Trapezium, on the left hand, for 35, in the foot of the said Trapezium, for the number of 30, you shall find in the common angle the Product to bee 17, and 30, whereof you must place the 30 under the first Denomination, and keeping the number 17 still in mind to be added to the next Product, multiply 35 into 7, and you shall find in the Trapezium the Product to be 4, and 5, whereunto if you adde the 17 which you had in mind, the Product will bee 4, and 22, whereof you must set the 22 under the fift Denomination, and keeping the 4 in mind, multiply againe the same 35 into 50, the Product whereof you shall find in the Triangle to be 29, and 10, whereto if you adde the 4 in mind, it will make 29, and 14, whereof the 14 is to bee placed under the fourth Denomination, and keeping the 29 in mind, multiply againe the said 35 into 31, and you shall find the Product thereof in the Triangle to be 18, and 5, whereunto if you adde 29 in mind, it will make in all 18, and 34, which 34 is to be set under the third Denomination, then keeping 18 in mind, multiply the foresaid 35 into 29, and you shall find the Product thereof in the Trapezium to be 16, and 55, whereunto if you adde the 18 in mind, the Product will be 17, and 13, which 13 is to be placed under the second Denomination, now because you haue gone through all the numbers of the Multiplicand, with the number 35 you must place

the

the 17. which you had in mind under the Denomination of minutes, and ſo having ended the ſecond Product, proceed to the finding out of the third Product by multiplying 10, firſt into 30, the Product whereof you ſhall find in the Trapezium, to be 5, and 0, whereof you muſt ſet the Cypher under the firſt Denomination, and keeping the 5 in mind, multiply againe 10 into 7, and you ſhall find the Product to be 1, and 10, whereunto if you adde the 5 in mind, the Product will be 1, and 15, whereof you muſt ſet the 15, under the fourth Denomination, and keeping the one in mind, multiply 10 into 50, and you ſhall find the Product thereof in the Trapezium to be 8, and 20, whereunto if you adde the 1 in mind, it will make 8, and 21, whereof you muſt ſet the 21, under the third Denomination, and keeping 8 in mind, multiply 10 into 31, and you ſhall find the Product thereof in the Trapezium to be 5, and 10, unto which if you adde the 8 in mind, the Product will be 5, and 18, whereof ſet downe the 18 under the ſecond Denomination, and keeping 5 in mind, multiply 10 into 29, the Product whereof you ſhall find in the Trapezium to be 4, and 50, whereunto if you adde the 5 in mind, the Product will be 4, and 55. Now becauſe you have gone through all the Numbers of the Multiplicand with 10, you muſt ſet downe the 4 which you had in minde under the Denomination of degrees, and ſo having the third Product, proceed to the fourth by multiplying 13 into 30, and you ſhall find the Product in the Trapezium to be 6, and 30, whereof ſet downe 30 under the fourth Denomination of Fractions, and keeping 6 in mind, multiply 13 into 7, the Product whereof you ſhall find in the Trapezium to be 1, and 31, whereunto if you adde the 6 in mind, the Product will bee 1, and 37, whereof ſet downe the 37 under the third Denomination, and keeping the 1 in mind, multiply 13 into 50, and you ſhall finde the Product in the Trapezum to bee 10, and 50, whereunto if you adde the 1 in mind, the Product will be 10, and 51, whereof ſet the 51 under the ſecond Denomination of Fractions, and keeping 10 in mind, multiply 13 in 31, the Product whereof you ſhall find in the Trapezium to be 6, and 43, whereunto if you adde the 10 in

mind the Product will bee 6, and 53, whereof set downe 53, under the first Denomination of Fractions, and keeping 6 in mind multiply 13 into 29, the Product whereof you shall find in the Trapezium to be 6, and 17, whereunto if you adde the 6 in mind, the Product will be 6, and 23, whereof set downe 23 under the Denomination of degrees. Now because you have gone through all the Numbers of the Multiplicand with the last Number of the Multiplyer, you must set downe the 6 which you had in mind under the Denomination of Integrums, that done, adde all the foure Products together, beginning on the right hand, saying thus, 30 and 0 is but 30, which set downe under the nethermost line, as you see in the former figure, then say 30 and 7 maketh 37, which you must set downe under the nethermost line next unto 30, then say 22 and 50 maketh 72, out of which by subtracting the Sexagenary Number, the remainder is 12, which is to be set under the line next unto the 37, keeping still the 60 in mind, which in this account maketh but one, and is to be added to the next ranke on the left hand, then say 1 in mind and 5 maketh 6, and 4 maketh 10, and 1 maketh 11, here set downe 1, and keepe the Article in mind, then say 1 in mind and 3 maketh 4, then 1 and 1 and 3 being added to 4 do make in all 9, out of which 9 you must subtract 6, which is but one 60, and is to be kept in mind and there remayneth 3, which is to be set downe by the 1, then say 1 in mind and 7 is 8, and 1 is 9, and 4 is 13, and 9 is 22, here set downe 2, and keepe the two Articles in mind, then say 2 and 3 maketh 5, and 2 is 7, and 5 is 10, and 2 maketh 12 tens, which do make 2 sixties, and are to be kept in mind, then say 2 and 1 maketh 3, and 8 maketh 11, and 3 maketh 14, wherefore set downe 4, keeping the 1 Article in mind, then say 1 in mind and 5 is 6, and 1 is 7, and 1 is 8, then take 6 from 8 and there remayneth 2 which set downe by the 4, and keepe one sixty in mind, then say 1 in mind and 3 is 4, and 5 is 9, and 7 is 16, wherefore set downe 6, and keeping one in mind, say that 1 and 5 is 6, and 5 is 11, and 1 is 12, which maketh two sixties to bee kept in mind, then say 2 in mind and 3 is 5, and 4 maketh 9, which set downe,

then

then say 2 is 2, which set downe by the 9, under the Denomination of degrees, then say 6 and nothing is 6, which set downe under the Denomination of Integrums, for that 6 in this place signifieth 6 sixties, which is in value 360 degrees, and being divided by 30, because 30 degrees make the one whole signe, you shall find in the Quotient 12 signes, so as the totall summe of the soure Products is 12 signes, 29 degrees, 6′, 24″, 2‴, 31⁗, 12ᵛ, 37ᵛⁱ, 39ᵛⁱⁱ, as the former Example sheweth, and this is the very same Example which I wrought before when I taught you how to multiply Astronomicall Fractions according to Gemma Frisius his Rule, and they both do wholly agree in every condition, saving that to worke by the Sexagenary Table is the readier way of the two.

An Example how to divide Astronomicall Fractions, by help of the Sexagenary Table.

SUppose then that you would divide the former totall Sum or Product found by Multiplication, which is 6 Integrums, 29 degrees, 6′, 24″, 2‴, 31⁗, 12ᵛ, 37ᵛⁱ, 39ᵛⁱⁱ, by this Divisor 29 degrees, 31′, 50″, 70‴, 30⁗, which in the former Example of Multiplication, was the Multiplicand. Now to Divide these two Numbers the one by the other, you must do thus; first you must set downe the row of Denominations, as you did before in Multiplication, and next under that the Dividend, then right under that the Divisor, and on the right hand behind a crooked line made like a halfe Moone, all the severall Quotients are to be set one by another in a right line, as you may see by the figures hereafter following.

Dedomination	In.	De					v	vi	vii
The Didend	6	29	6	24	2	33	12	37	39
The Divisor		29	31	50	7	29			

Heere having made your row of Denominations and set down your Dividend, you have to consider whether the first Number of your Divisor, bee greater than the first Number of your Dividend, for if it be, then you must place your Divisor one space further towards

your right hand, as in this Example, because the first Number of your Divisor 29. cannot be taken out of 6, you set it under the second Number of the Dividend, and so all the rest of the Numbers successively towards the right hand, as the former Example sheweth. Now the order of working is thus: You must first seeke out the first Number of your Divisor, in the first Collum of the Trapezium on the left hand, which is 29, then in the row right against that 29, on the right hand, you have to seeke out those Numbers of the Dividend, which do stand right over the first Number of your Divisor, which is 6, and 29, and if you cannot find those Numbers justly, then seeke in the selfe-same row a Number, which is somewhat lesse and nearest in value unto it, as in this Example, because you cannot find 6, and 29, you take 6, and 17, the Quotient whereof you shall find in the foot of the Trapezium right under the said Number 6, and 17, to be 13 degrees, which you must set in the Quotient Line, and that is your first Quotient, having his proper Denomination over his head, which are degrees, and are to be found by the Rule before taught. Thus you see here that the Number of the Divisor is to be found in the outermost Collum on the left hand, and the Number of the Dividend in that row which is right against the said Divisor, and the Quotient in the foot of the Trapezium, right under the Number of the Dividend last found. Then you have to multiply the whole Divisor (that is to say) every particular Number thereof by the first Quotient 13, which you may do by helpe of the Table, as you did before in the Example of Multiplication, and is to be set downe in this manner. And you shall find the whole Product to be 6.23.53.5.137.30.

29.31.50 7.30.
13

Which being set right under the first Dividend, is to be subtracted out of the same, and the remainder to be written over the head of the Dividend, as you do in common division, first then to multiply all the particular Numbers of the Divisor by the Quotient 13, and to find every Product thereof, resort to the Table and seeke for 13 in the foot of the Trapezium, and for 30, which is the multiplicand in the outermost Collum of the Trapezium

pezium, on the left hand, and the common angle will shew the Product which is 6 and 30, whereof you must set downe 30 under 30, and keeping 6 in mind, multiply againe 13 into 7. the Product whereof you shall finde to be 1 and 31, unto which adde the 6 which you kept in mind, and the Product shall be 1 and 37. where you must set down 37, and keeping 1 in mind, multiply againe 13 into 50, and you shall find the Product to be 10 and 50, whereuntoif you adde 1 in mind, the Product shall be 10 and 51, wherof you must set down 51, and keeping 10 in mind, multiply 13 into 31, the Product whereof you shall finde in the Trapezium to be 6 and 43, whereunto if you adde the 10 which you had in mind, the Product shal be 6 and 53, wherof set down 53 and keeping 6 in mind, multiply 13 into 29 and you shall finde the Product to 6 and 17, whereunto if you adde the 6 in minde, the Product shall be 6 and 23, whereof set downe 23 under the last Number of your Multiplicand, and because you have no more Numbers of the Divisor to be multiplyed, set downe 6 in minde on the left hand, so shall the whole Product be 6, 23, 53, 51, 37, 30, as the former Example sheweth, which Product is to be subtracted out of the first Dividend, and the remainder is to be set down over the head of that Dividend, as you see in this Example next following, wherein the first Dividend is first set downe, and right under that the foresaid Product, which is the first Product, and the remaynder above the Dividend, and the Quotient 13, is set in the Quotient line, which is your first Quotient.

	29	31	50	7	30				The Divisor.
					13				The 1. Quotient.
6	23	53	51	37	30				The 1. Product.

The Remaynder.		5	12	32	25	1			
The first Dividend.	6	29	6	24	2	31	12	37	30
The first Product.	6	23	53	51	37	30			

(13.

And remember (in making your Subtraction) to begin with the first Number of the foresaid Product which is on the right hand, and when you cannot take it out of the Number standing right over his head, to borrow alwayes 60 of the next Number

on the left hand, and to pay it home againe with 1, for there 60 is but 1. This done, remove your Diviſor one ſpace further towards the right hand (that is to ſay) ſet the firſt Number of your Diviſor under 12, which is the ſecond Number of the ſecond Dividend, which together with the firſt remainder is 5, 12, 32, 25, 1, 12, 37, 30, and all other Numbers of the Diviſor orderly towards your right hand, as you ſee in this Example,

The ſecond Dividend.	5	12	32	25	1	12	37	30
The Diviſor.		29	31	50	7	30	1	

Then aſke how many times 29 is in 5 and 12, which Number you muſt ſeeke in the Trapezium in the row that anſwereth towards the right hand, to the firſt number of your Diviſor which is 29, ſtanding in the outermoſt Collum of the Trapezium on the left hand, and becauſe you cannot finde 5 and 12 in that row, you may ſeeke in the ſame row the Number which is nigheſt unto it, but leſſe, which you ſhall finde to be 4 and 50, and right under that in the foot of the Trapezium you ſhall finde 10, which muſt be your ſecond Quotient, by which Quotient you have to multiply all the particular Numbers of the Diviſor, in ſuch order as is before ſet downe, and you ſhall finde the Product of that Multiplication to be 4, 55, 18, 21, 15, 0, which Product you muſt place under the ſecond Dividend, ſetting 4 under 5, and 55 under 12, and ſo forth, orderly towards the right hand, that done, ſubtract the ſame Product out of the Numbers of the ſecond Dividend, ſtanding right over the ſaid Product, and the remainder will be 17, 14, 3, 46, 12, as you ſee in this Example.

The remaynder.	17	14	3	46	12	37	30	
The ſecond Dividend.	5	12	32	25	1	12	37	30	(13. 10.
The ſecond Product,	4	55	18	21	15	0		

Now remove your Diviſor one ſpace further towards the right hand, by ſetting the firſt Number of your Diviſor, which is 29 under 14, which is the ſecond Number of the third Dividend, and ſo all the reſt orderly towards the right hand, as you ſee in this Example.

Then

Then aske how many times 29 is contained in 17 and 14, which Dividend, because you cannot find it in the row that answereth to that 29 which standeth in the outermost Collum of the Trapezium, on the left hand (for all those numbers are too little) you must seke for it in that Collum which standeth right upon 29 in the foot of the Trapezium, neither shall you find it there, but you shall find 26 and 55 which is somewhat lesser, yet in value nigheſt unto 17 and 14, and right against that, you shall find in the outermost Collum on the right hand 35, which must bee your third quotient, to be set in the quotient line, whereby you have againe to mutiply the whole Divisor in such order as before, the product whereof you shal find to be 17, 13, 34, 14, 22, 30, which product being placed under the Dividend, by setting 17 under 17 and 13 under 14, and so forth orderly towards the right hand, subtract the said product (beginning at the right hand) out of the numbers which stand right over the said product, and the remainder wil be 29, 31, 50, 7, 30, which is to be set downe over the third Dividend, and the rest to be cancelled, as you see in this example.

The 3 Dividend.	17	14	13	46	12	37	30	(13, 10, 35,
		29	31	50	7	30		
The remainder.								
The 3 product.	17	13	14	14	22	30		

And now the remainder of the third Dividend is come to be the fourth Dividend: wherfore remove your Divisor one space further towards the right hand, by setting 29 under 29, and the rest orderly towards the right hand, as you see in the example following: then aske how many times 29 is contained in 29, which being but once, your fourth quotient is 1, and is to be set in the quotient line, whereby the whole Divisor being multiplyed, the product will be 29, 31, 50, 7, 30, which you must place under the fourth Dividend; and being subtracted out of the same, nothing will remaine, and so the whole quotient will be 13 degrees, 10′, 35″, 1‴, as you see in this example.

The fourth Dividend.	0	0	0	0	0		degr. ′ ″ ‴
The fourth Product.	29	31	50	7	30		(13.10.35.1.
	29	31	50	7	30		

Another example of Division.

If the daily motion of the Moone be 13 degrées, 3′, 53″, 56‴, 23⁗, 58ˣ, in what time shall she make her whole revolution, allowing 360 degrées to that revolution, which is otherwayes called the Month of Paragration. Here, for so much as this example is to be wrought by Division, and that your Dividend is a simple and whole Number (that is to say) 360 degrées, without any Fractions of divers denominations annexed thereunto: you must first set down the 360 degrées, and next to that, towards your right hand, set downe a long row of Cyphers, with so many denominations over their heads as you shall think needfull to serve your turne, and right under your Dividend set your Divisor, as you see in this example.

The Denominations.	Inte.	De.	′	″	‴	⁗	v	vi	vii	viii	ix	
The Dividend.	360	0	0	0	0	0	0	0	0	0	c	(27.
The Divisor.		13	3	53	56	23	58					
The 1 Product.	352	45	16	22	47	6						

Here you first must aske how many times 13 is in 360, and by the common Rule of Division you shall find the Quotient to be 27, which you must set downe in the Quotient line, and by that Quotient you have to multiply every particular number of the Divisor, beginning on the right hand, as you do in Multiplication, saying, that 27 times 58 is 26 and 6, as the Trapezium sheweth, for by seeking 27 in the foot of the Trapezium, and for 58 in the outermost Columne on the right hand, you shall finde in the common angle 26 and 6, wherefore set downe 6 under 58, and keeping 26 in mind, multiply 23, which is the second number of the Divisor by 27, and by the Trapezium you shall find the Product thereof to be 10 and 21, whereunto if you adde the 26 in mind, it will make 10 and 47, whereof set downe

47,

47 under 23, and keeping 10 in mind, multiply 56 which is the third number of the Diuisor by 27, and in the Trapezium you shall find the product to be 25 and 12, whereto if you adde the 10 in mind, it will make 25 and 22, whereof set downe 22 under 56, and keeping 25 in mind, multiply 53 which is the fourth number of the Diuisor by 27, the product whereof you shal find by the Trapezium to be 23 and 51, whereto if you adde 25 in mind, it will make 76 which is one 60 and 16, whereof set down 16 under 53, and adde the one in mind to 23, and that will make 24, which you must keepe in mind, then multiply 3 which is the fift number of the Diuisor by 27, the product whereof you shal find in the Trapezium to be 1 and 21, whereto if you adde the 24 in mind, it will make 1 and 45, whereof set downe 45 under 3, and keeping one in mind, multiply 13 which is the last number of the Diuisor by 27, the product whereof you shall find in the Trapezium to be 5 and 51, whereunto if you adde one in mind, it will make 5 and 52, here because the first 5 is 5 sixties, it maketh in all 352 and is to be set under 360, so as the first product of this Multiplication containeth these numbers, 352 45, 16, 22, 47, 6, as you see them set downe in the former example, and this product is to be subtracted out of 360 which is the first Dividend. And to avoyd confusion, it shall not be amisse to set downe the first Dividend and the first product apart by themselves thus.

The 1 remainder.	7	14	43	37	12	54	0	0	0	
The 1 Dividend.	360	ø	ø	ø	ø	ø	ø	ø	ø	(27,
The 1 Product.	352	45	16	22	47	6				

Here beginning on the right hand, say thus: take 6 out of nothing, which will not bee, wherefore you must borrow 60, then by taking 6 out of 60, there will remaine 54, which you must set above that Cypher which standeth right over 6, and cancell the 6, keeping still the one 60 which you borrowed in mind, then say, 47 and one in mind maketh 48, which will not be taken out of nothing, and therefore you must borrow againe one 60, as you did before, so shall the remainder be 12, which is to be set above the Cypher which standeth right over

47, and

47, and cancell 47, and so proceed with like order in subtracting all the rest of the numbers of the first Product out of the first Dividend, so shall the remainder be 7, 14, 43, 37, 12, 54, as you see them set down in the former figure. Now having to remove your Divisor one space further towards the right hand, you shal do well to make your first remainder, which is, 7, 14, 43, 37, 12, 54, to be your second Dividend, and under that to set your Divisor, as you see in this example.

The 2 Dividend.	7 | 14 | 43 | 37 | 12 | 54 | 0 | 0 | 0	(27. 33.
The Divisor.	|13| 3 |53|56|23|58| 1	

Then aske how many times 13 is contained in 7 and 14, and having found 13 in the foot of the Trapezium seek in that Columne for 7 and 14, and not finding it there, take in the self-same Columne that number which is nighest in value unto it, and lesse, which you shall finde to be 7 and 9, right against which, in the outermost Columne on the right hand is 33, which must be your second Quotient, and is to be set in the Quotient line next unto 27, and by this Quotient you have to multiply every number of the Divisor as you did before, the Product whereof you shall finde to be 7, 11, 8, 40, 1, 10, 54, wherfore you must first set downe your second Dividend, and then the second Product right under the same, as you see in this example.

The second remainder.	1 | 3 | 34 | 57 | 11 | 43 | 6 | 1 | 1	(27. 33.
The 2 Dividend.	7 | 14 | 43 | 37 | 12 | 54 | 0 | 0 | 0	
The 2 Product.	7 | 11 | 8 | 40 | 1 | 10 | 54 | 1 | 1	

Which Product being subtracted out of the second Dividend, the remainder will be 3, 34, 57, 11, 43, 6, which is to be set above the second Dividend, and the Product to be cancelled, as you see in the former example. Here having to remove againe your common Divisor one space further towards the right hand, set downe first the last remainder, which now must be your third Dividend, and under that set the common Divisor, as you see in this example.

The 3 Dividend.	|3|34|57|11|43| 6| 0|0|0	(27. 33. 16.
The Divisor.	|13|3 |53|56|23|58| 1	

Then

Then aske how may times 13 is contained in 3 and 34, here by seeking in the foot of the Trapezium for 13, though you cannot finde in that Columne 3 and 34, yet you shall finde 3 and 28 which is the nigheſt, right against which in the outermost Columne on the left hand, you shall find 16, which must be your third Quotient, and is to be set in the Quotient line, by which Quotient you must multiply the whole Diviſor as before, the Product whereof you shall finde to be 3,29,2,23,2,23,28, which being the third Product, you must set downe under the third Dividend which was your last remainder, and to be subtracted out of the same, as you see in this example.

The third Dividend.	1	5	54	48	40	42	32	0	0
	3	3	4	5	7	2	4	8	8	8	8
The third Product.	3	29	2	23	2	23	28		

And the remainder will be 5,54,48,40,42,32, which you must set above the third Dividend, and all the inferior Numbers are to be cancelled, as you see in the former example. Here having againe to remove your Diviſor one space further towards your right hand, the last remainder must be your fourth Dividend, under which the common Diviſor is to be set thus.

The 4 Dividend.	7	54	48	40	42	32	0	0	dayes , " " "
The Diviſor.	1	1	3	53	56	23	58		(27	33	16	26

Here asking how many times 13 is contained in 5 and 54, seeke for 13 in the foot of the Trapezium, in whose Columne you shall not finde 5 and 54, but 5 and 51, which is nigheſt unto it, and right against that, in the outermost Columne on the left hand, you shall find the Quotient to be 27, by which if you should multiply the whole Diviſor, the whole Product thereof would be 5,56,21,24,1,47,6, which is more than the Dividend, and therefore you must make your Quotient one leſſe, setting downe no more but 26 in the Quotient line, by which if you multiply the Diviſor, the Product will be 5,26,41,22,25,14,8, which being subtracted out of the fourth Dividend, the remainder will be 28,7,18,17,52, which remainder if you will

you

you may make to be a first Dividend, and then to worke as before, if you would have your Quotient to extend to smaller denominations, which I leave to do, because I thinke that thirds be small enough. And as often as the Product of any particular Quotient shal be greater than the Dividend, remember to take a lesse Quotient, even as you do in common Division. But now you have to note, that though this whole Quotient here signifieth time, for the first Quotient signifieth dayes, the second Quotient minutes of dayes, the third Quotient seconds, & the fourth Quotient thirds: yet for so much as the day is to be counted by 24 houres, and not by minutes, you must therefore reduce all the particular Quotients, saving the first, into houres and parts of houres, even to so small denominations as you shall thinke good your selfe. by helpe of this Rule, which in Division of Astronomicall Fractions, biddeth to multiply the Quotient by 2, and to divide the Product thereof by 5. As here, if you multiply the 33, which is the second Quotient by 2, the Product wil be 66, which being divided by 5, the Quotient will bee 13 houres, and one 60 remaining to be kept in mind: wherefore set downe in the place of 33', 13 houres, then multiply the third Quotient, which is 16 by 2, and that will make 32, whereunto if you adde the one 60 in mind, it will make in all 92, which being divided by 5, the Quotient will be 18, and two sixties, which is 120 remaining to be kept in mind: wherefore instead of 16'' which was the third Quotient, set 18' of an houre, and then proceed to the fourth Quotient, which is 26''', which being multiplyed by 2 maketh 25, whereunto if you adde 120, it will make 172, which if you divide by 5, the Quotient will be 34'' of an houre, which is to bee set downe in the place of 26, so shall your whole former Quotient containe 27 dayes, 13 houres, 11' of an houre, and 34'' of an houre. And note, that as in the Division of Astronomicall Fractions, to bring the Quotients to like denomination, you do multiply by 2, and divide by 5, so in Multiplication, to reduce the numbers of the Multiplyer, being of divers denominations, to one selfe denomination, you must by order reverse, multiply by 5, and divide by 2, whereof I have given you an example before,

whereas

whereas I shew you how to multiply Astronomicall Fractions, according to Gemma Frisius, without the helpe of the Sexagenarie Table.

An Example shewing how to extract the square Root out of Astronomicall Fractions, by helpe of the Sexagenarie Table.

SUppose the number given to be 17′,12″,33‴,4⁗, whereof you have to take the square Root, here having set downe the said numbers, with the proper Denominations over their heads, as you see in the example following, first set a prick under the last number on the right hand, and then prick every other number, leaving one void space betwixt every two pricks, as you do when you seeke the square Root of whole numbers or Integrums, as you see here in this example. ′ ″ ‴ ⁗

The number given, 17.12.33.4.

Then resort to the Table, and seeke amongst the Products which are placed next to the line A.B. as well in the Trepezium, as in the Triangle, & see whether you can find the number standing over the first prick on the left hand, which is 17, and 12, but not finding it there, you must take that which is nigheſt unto it, and lesse, which you shall find in the Triangle towards B, to be 17,4, the Root whereof you shall find both in the head, and also in the outward Columne of the Triangle on the right hand to bee 32, answering to the foresaid square Root, which Root you must place behind the Quotient line, then subtract 17 and 4, out of 17, and 12, and there will remaine 8, which is to be set over 12, and the 17, & 12, to be cancelled, that done, double the Root 32, which will be 64, that is to say, 1 and 4, and setting 1 unde 8, and 4 under 33, aske how many times 1 is in 8, and there is 8, which is the second Quotient, by which you must multiply 1 and 4, the Product whereof will be 8, and 32, which being subtracted from the upper number, which is 8, and 33, the remainder is 1, which is to be set over 33, and the 8, and 33 to be cancelled. Finally, multiply the second Quotient 8 in it selfe, and the product will be 1, and 4, which being subtracted out of the former remainder of

the

the given Number nothing remayneth, so you shall find 32′, 8″, to be the Root of the given Number, as this Example sheweth.

Which Root if you multiply into it selfe squarely, the Product will be like unto the Number given, and by this meanes you shall find the given Number to be a just square Number. But you have to note, that if the last Denomination standing on the right hand be odde and not even, as thirds, or fifts, then you must set downe a Cypher beyond the last Denomination towards your right hand, and under that Cypher set your first pricke, as in this Example 3′, 8″, 9‴, 17⁗, 3ᵛ, here because the last Denomination on the right hand is odde, that is to say fifts, you must therefore set down next unto it towards thy right hand a Cypher having over his head the next Denomination which is vi, and is even, wherfore set your first pricke under that Cypher and so proceed towards your left hand pricking every other Number, and then worke as followeth.

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      ″   ‴
      8   1
                ⁗
1 7 1 2 3 3 4 . (32′.8″.
      1   4 4
          1.
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First then seeke in the Table amongst the square Numbers nigh unto the Line A, B, for 3 and 2, which should stand in one selfe square Angle next unto the line A. B. for those only are square Numbers, and not finding it amongst the square Numbers, take that square which is nighest unto it and lesse, which you shall find to be 2, and 29, the Root whereof is 13, and to be found as well in the foot of the Trapezium right under the fore-said square Number, as also in the outermost Collum of the said Trapezium on the left hand standing right against the same square Number, which Root is to be set in the Quotient line, that done, subtract the foresaid square Number out of the Number standing right over the last prick on the left hand which is 3, and 2, and the remainder will be 13, which is to be set over 2, and the 3, & 2 to be cancelled, as you see in the former example, that done, double the foresaid found Root 13, which will make 26, by which you have to divide 13, and 9, wherefore resort to the

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      ″   ‴
      13.   ᵛ ⁵ᵛⁱ
      ⁴ ⁴ .9.17.8.0 (13.
      2.42.
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Tra-

Trapezium, in the foot whereof you shall find 26, in whose Collum not finding 13 and 9, take that which is nighest unto it and lesse, which is 13 and 0, right against which you shall find in the outermost Collum on the right hand 30, which should bee your Quotient, but because it is too great, and that the Square thereof cannot be taken out of the remaynder, you must make the Quotient one lesse, and set in the Quotient Line no more but 29, by which 29 if you multiply 26, the Product will be 12, and 34. as the Trapezium sheweth, which being subtracted out of 13, and 9. standing over 16, the remaynder will be 35, which 35 is to be set over 9, then by multiplying the second Quotient which is 29 into it selfe, you shall find by the Trapezium the Product to be 14 and 1, which being subtracted out 35, and 17, the remaynder will be 21 and 16, which is to be set over 35 and 17, as you see in this Example.

That done, double both the Quotients or whole Root, and the Product will be 26, 58, and having set 58 under 8. and 26 under 16, divide 21, 16, and 8 by 26, 58, wherefore looke for 26 in the foot of Trapezium, and for 21, 16, in that Collum, and not finding it there, take that which is nighest unto it and lesse, which you shall find to be 21, and 14, and right against that in the outermost Collum on the right hand standeth 49, which being too great to be your Quotient by 2, therefore make your Quotient no more but 47, which being set downe in the Quotient line, multiply thereby 26, 58, and the Product will be 21, 7, 26, which being subtracted out of 21, 16 and 8, the remaynder will be 8 and 42, then setting 42 over 8, and 8 over 16, cancell 21, 16 and 8, then multiply 47 into it selfe squarely, and you shall find the Product thereof in the Triangle to be 36, 49, which being subtracted out of 8, 42, and 0, the remainder will be 8, 6, 11, as you see in this Example.

21
~~1~~ ~~3~~ ~~3~~ ~~5~~ 16
~~3~~. ~~2~~. ~~9~~ ~~1~~. ~~7~~. 80. (13. 29.
~~2~~ ~~4~~ ~~9~~ 26 The double of the 1 Quo.
14. 1 The Square of the 2 Quo.

And

And thus you have found the Root of your given Number to bee 13, 29, and 47, and whether you have done right or not you may know by multiplying the said Root into it selfe squarely, and by adding to the Product thereof the remainder, and if you would find a more exact Root, then adde more Cyphers to your given Number towards your right hand, together with their Denominations over their heads, as they increase successively, and bee sure alwayes that the Numbers of the Cyphers be even and not odde, as 2,4,6, and so forth, and the more Cyphers that you adde to the given Number (so as the Number of them be even) the more exact Root you shall have.

Now to give to every Number of the Root his proper Denomination, do thus, take halfe of those Denominations that are over every prick, and those shall be the Denominations of the Root, as in the former Example, whereas Seconds do stand over the last prick on the left hand, take halfe of that which is 1", wherefore the first Number of your Root must be minutes, and over the rest of the Numbers of the Root, set Seconds, Thirds, and so forth, successively untill you have brought your Root to as small Denomination as you desire.

FINIS.

A briefe

DESCRIPTION

of the Tables of three ſpeciall Right Lines belonging to a Circle, called

SINES,
TANGENTS, and
SECANTS.

Not only ſhewing why they were firſt invented, and defining the proper tearmes thereto belonging:

But alſo ſhewing divers neceſſary Uſes thereof.

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